When Is Employment a Favor?

Abstract

Traditional labor theory views employment as a mutually beneficial exchange where compensation equals an employee's marginal contribution in equilibrium. Yet in practice, elite firms sometimes retain workers on terms that appear generous relative to observed marginal product. Interpreting such cases as "favor" can be plausible but is also theoretically risky because wages can rationally exceed contemporaneous marginal product under search frictions, vacancy delays, training costs, relational contracts, morale constraints, and fair-wage concerns. We develop a microfounded dynamic framework that derives a *wage band*: a firm-value-maximizing lower bound to satisfy retention (outside options, morale, and fairness) and an upper bound that internalizes replacement costs, vacancy delays, training, and organizational externalities. We then define *Economic Favor* as compensation that exceeds this rigorous upper bound and show conditions under which it is strictly positive. The framework delivers sharp comparative statics for replaceability and scarcity, an explicit decomposition of observable wage gaps, and a measurement strategy with implementable bounds. We provide proofs, identification guidance, and proposed evolution for immediate reproducibility. The result is a concept of "favor" that is theoretically disciplined, empirically testable, and insulated from standard objections.

1 Introduction

Standard models often equate pay with a worker's marginal revenue product (Becker, 1962; Milgrom & Roberts, 1992). "I am earning pay based on how productive I am, the company is not doing me a favor." However, contemporary evidence from world-class firms with high market wages shows that an employee's productive output relative to all they earn in wages, benefits, and other allowances may be lower than what the same employee could command elsewhere for the same productive output (Manning, 2003; Mortensen & Pissarides, 1994). This means that, on a per-dollar basis, the employee is receiving more favorable terms than they would in the broader market. The employee is not unique; their type may be found if the company chooses to replace them. The employee benefits much more from the company than the company benefits from them, at least relative to what is generally obtainable in the market. The employee thus needs the firm much more than the firm needs the employee, simply because the employee type is not unique and can be replaced. Yet the world-class firm chooses to retain the employee, implying that the firm is extending a form of favor, since it could obtain a higher-productivity employee per dollar, with limited institutional cost, given its ability to attract high-caliber talent (Oyer & Schaefer, 2011).

On the other hand, such firms also provide possibilities that show *persistent* wage premia in settings with rare skills, team production, retention frictions, vacancy delays, employee replacement costs, training costs, fair-wage or morale constraints, and durability of match-specific capital (Shapiro & Stiglitz, 1984; Lazear, 2000). In these environments, wages *can* rationally exceed a static measure of marginal product without implying managerial altruism or inefficiency.

This paper proposes a disciplined answer to a simple but contentious question: *When, if ever, does employment constitute a "favor" from firm to employee?* We argue that the only sensible definition must be *operational*, not naive. Specifically, we first derive a *wage band*—a closed interval of value-maximizing wages—from a dynamic model with search, replacement, training, vacancy delay, retention, and fairness constraints. We then define *Economic Favor* as compensation above the *upper* bound of that band. This definition ensures that efficiency wages, retention premia, and relational contracts are not mislabeled as favors.

2 Environment and Timing

Consider an infinite-horizon discrete-time firm–worker match with discount factor $\delta \in (0, 1)$. In each period:

1. The firm offers wage w_t to incumbent employee i.

2. The worker compares w_t with her outside option b_t (distribution depends on scarcity) and a fair-wage/morale benchmark φ_t.[1]

3. If the match continues, the firm obtains period revenue v_t from i (marginal contribution net of team spillovers not attributable to others). If the match ends, the firm incurs replacement costs: a vacancy delay (expected ℓ periods) with foregone output λ_v per

[1] The φ_t term can summarize internal equity, reputation, or relational norms that affect effort and quit propensity; we keep it reduced form to remain agnostic about microfoundations (e.g., fair-wage, gift exchange, or internal equity).

period, direct hiring cost K, and expected training cost H prior to reaching steady-state productivity.

Let $R \in [0,1]$ denote *replaceability* (higher R means easier replacement: shorter ℓ, lower K, lower H) and $S \geq 0$ denote *scarcity* (higher S means higher outside options b_t and tighter markets). We assume stationarity for the main results; dynamics are treated in Section 8.

3 Value-Maximizing Wage Band

We start by deriving a lower and upper bound for value-maximizing wages.

3.1 Retention lower bound

Let the retention constraint be

$$w \;\geq\; w_{\min} \;:=\; b + \phi(\varphi), \tag{1}$$

where b is the worker's outside option and $\phi(\varphi) \geq 0$ is the minimum premium consistent with morale/fairness/relational constraints delivering the effort and quit hazard the firm expects.[2]

3.2 Replacement upper bound

If the firm contemplates replacing the worker, the expected value loss from replacement is:

$$\underbrace{\ell \cdot \lambda_v}_{\text{vacancy output loss}} + \underbrace{K}_{\text{hiring}} + \underbrace{H}_{\text{training ramp-up}}. \tag{2}$$

Let $\Delta(R)$ denote the *total* replacement wedge: $\Delta(R) := \ell(R)\lambda_v + K(R) + H(R)$, with $\Delta'(R) < 0$.

Let $\bar{w}$ denote the wage the firm expects to pay to a replacement at steady state.[3] Then the firm will choose to *retain* the incumbent whenever

$$v - w \;\geq\; -\Delta(R) + v - \bar{w}, \tag{3}$$

or equivalently

$$w \;\leq\; w_{\max} \;:=\; \bar{w} + \Delta(R). \tag{4}$$

[2]One can microfound ϕ from an efficiency-wage model, internal equity, or relational contract self-enforcement constraint; our reduced form nests those cases.

[3]This can be the market wage for equivalent skill conditional on R, S, i.e., $\bar{w} = \mathbb{E}[b] + \text{market premium}$; empirically, it can be measured from job-level offers or internal HR data.

Thus $w_{\max}$ is the *largest* value-consistent wage: above it, replacement strictly dominates retention in firm value.

3.3 The wage band and Economic Favor

Definition 1 (Value-Maximizing Wage Band)**.** *Given* (b, φ, R, S)*, the value-maximizing wage band is*

$$\mathcal{W} \;:=\; \big[w_{\min},\; w_{\max}\big] \;=\; \big[b+\phi(\varphi),\; \bar{w}+\Delta(R)\big].$$

Any wage $w \in \mathcal{W}$ *weakly maximizes firm value given observables and constraints.*

Definition 2 (Economic Favor)**.** *Given* $\mathcal{W}$*, the* Economic Favor *is the nonnegative transfer*

$$F^E \;:=\; \max\{\,0,\; w - w_{\max}\,\}.$$

By design, F^E is *only* positive when compensation exceeds the *upper* value-consistent bound. This definition rules out mislabeling efficiency wages, fair-wage premia, or retention premia as "favors."

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Consider an infinite-horizon discrete-time firm–worker match with discount factor $\delta \in (0,1)$. In each period:

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5.1 Retention lower bound

The retention constraint is

$$w \;\geq\; w_{\min} \;:=\; b + \phi(\varphi), \tag{5}$$

where b is the worker's outside option and $\phi(\varphi) \geq 0$ is the minimum premium consistent with morale/fairness/relational constraints delivering the effort and quit hazard the firm expects.

5.2 Replacement upper bound

If the firm contemplates replacing the worker, the expected value loss from replacement is:

$$\underbrace{\ell \cdot \lambda_v}_{\text{vacancy output loss}} + \underbrace{K}_{\text{hiring}} + \underbrace{H}_{\text{training ramp-up}} . \tag{6}$$

Let $\Delta(R)$ denote the *total* replacement wedge: $\Delta(R) := \ell(R)\lambda_v + K(R) + H(R)$, with $\Delta'(R) < 0$.

Let $\bar{w}$ denote the wage the firm expects to pay to a replacement at steady state.[5] Then the firm will choose to *retain* the incumbent whenever

$$v - w \;\geq\; -\Delta(R) + v - \bar{w}, \tag{7}$$

or equivalently

$$w \;\leq\; w_{\max} \;:=\; \bar{w} + \Delta(R). \tag{8}$$

Thus $w_{\max}$ is the *largest* value-consistent wage: above it, replacement strictly dominates retention in firm value.

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Definition 3 (Value-Maximizing Wage Band)**.** *Given* (b, φ, R, S)*, the value-maximizing wage band is*

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[5] This can be the market wage for equivalent skill conditional on R, S, i.e., $\bar{w} = \mathbb{E}[b] + \text{market premium}$; empirically, it can be measured from job-level offers or internal HR data.

Definition 4 (Economic Favor)**.** *Given $\mathcal{W}$, the* Economic Favor *is the nonnegative transfer*

$$F^E \;:=\; \max\{\,0,\; w - w_{\max}\,\}.$$

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6 Formal Propositions on Favor Dynamics

Proposition 1 (Favor Exists When Employee Dependence Exceeds Firm Dependence)**.** *If the employee depends on the firm more than the firm depends on the employee, then retaining the employee with above-productivity compensation constitutes an Economic Favor.*

Proposition 2 (Replaceability Reduces Favor)**.** *Favor diminishes as employee replaceability increases across the firm:*

$$\frac{\partial F_i}{\partial R_i} < 0.$$

Proposition 3 (Scarcity Amplifies Favor)**.** *The scarcer the job opportunity, the greater the favor extended by the firm when the employee's compensation per unit of productivity exceeds what is obtainable in the broader market:*

$$\frac{\partial F_i}{\partial S_i} > 0.$$

Proposition 4 (Favor Persists Over Time if Compensation Outpaces Productivity)**.** *Let*

$$F_i(t+1) = F_i(t) + \Delta W_i(t) - \Delta V_i(t).$$

If $\Delta W_i(t) \geq \Delta V_i(t)$, then

$$F_i(t+1) \geq F_i(t),$$

so favor persists or grows when compensation growth outpaces productivity growth.

Proposition 5 (No-Favor Region)**.** *If $w \in [w_{\min}, w_{\max}]$, then $F^E = 0$. In particular, $w > v$* does not *imply $F^E > 0$ whenever $w \leq \bar{w} + \Delta(R)$.*

Proof. Immediate from Definition 4 and the construction of $w_{\max}$. Efficiency wages or other premia that keep w within $\mathcal{W}$ are value-consistent. □

Remark 1 (Naive Gap vs. Economic Favor)**.** *Define the naive gap $F^A := w - v$. Then $F^A > 0$ can arise* within $\mathcal{W}$ *due to $\Delta(R)$, $\bar{w}$, and $\phi(\varphi)$. Only $w > w_{\max}$ triggers $F^E > 0$.*

7 Comparative Statics: Replaceability and Scarcity

We link R (replaceability) and S (scarcity) to the wage band.

Assumption 1 (Monotonicities)**.** $\Delta'(R) < 0$ *(easier replacement reduces wedge);* $\bar{w}'(S) > 0$ *and* $b'(S) > 0$ *(scarcity increases outside and market wages); and* $\phi'(\varphi) \geq 0$ *(stricter morale/fairness raises the lower bound).*

Proposition 6 (Comparative Statics)**.** *Under Assumption 1:*

$$\frac{\partial w_{\max}}{\partial R} = \Delta'(R) < 0, \qquad \frac{\partial w_{\max}}{\partial S} = \bar{w}'(S) > 0,$$
$$\frac{\partial w_{\min}}{\partial S} = b'(S) > 0, \qquad \frac{\partial w_{\min}}{\partial \varphi} = \phi'(\varphi) \geq 0.$$

Hence scarcity shifts the band upward; replaceability tightens the upper bound downward.

Proof. Direct differentiation of (5)–(8). □

Corollary 1 (Favor Likelihood)**.** *For fixed* w*, higher* R *reduces the chance that* $w > w_{\max}$*; higher* S *increases it. Thus* F^E *becomes less likely as replacement frictions ease.*

8 A Dynamic Version and Persistence

Let the firm adjust wages with convex cost $C(w_t - w_{t-1})$ (e.g., internal equity, renegotiation, menu costs). Define the *dynamic* upper bound:

$$w_{\max,t} \;:=\; \bar{w}(S_t) + \Delta(R_t) - \Gamma_t,$$

where $\Gamma_t \geq 0$ captures temporary value gains from a credible threat of fast replacement (e.g., in recessions). Suppose $w_t > w_{\max,t}$ at t. The firm solves

$$\min_{\{\Delta w_\tau\}_{\tau \geq t}} \sum_{\tau=t}^{\infty} \delta^{\tau-t} \Big\{ (w_\tau - w_{\max,\tau})_+ + C(\Delta w_\tau) \Big\}.$$

This yields gradual adjustment. A sufficient condition for persistence of F^E is that $C'(\cdot)$ is large relative to the per-period penalty of $w_\tau - w_{\max,\tau}$:

Proposition 7 (Persistence with Adjustment Costs)**.** *If* $C'(\cdot)$ *is sufficiently steep near* 0 *and* $w_{t-1} \gg w_{\max,t}$*, then* $F_t^E > 0$ *can persist for multiple periods even though the static upper bound is violated. As adjustment costs fall to zero,* F^E *is eliminated in one step.*

9 Decomposing Observed Wage Gaps

Observed wage-productivity gaps $w - v$ can be decomposed as

$$w - v = \underbrace{(\bar{w} - v)}_{\text{market premium}} + \underbrace{\Delta(R)}_{\text{replacement wedge}} + \underbrace{(w - \bar{w} - \Delta(R))_{+}}_{\text{Economic Favor}} + \underbrace{\phi(\varphi) + (b - \bar{w})}_{\text{retention/fairness misalignment}} . \quad (9)$$

Equation (9) shows that a positive gap is not informative about favor unless it exceeds $\bar{w} + \Delta(R)$.

10 Measurement and Identification Strategy

The framework is empirically implementable with HR/operations data:

- **Outside option** b: recruiting pipeline outcomes; realized outside offers; internal transfer offers.
- **Market wage** $\bar{w}$: posted wage distributions for equivalent roles; external compensation surveys.
- **Replacement wedge** $\Delta(R)$: (i) time-to-fill ℓ; (ii) vacancy output loss λ_v (role-level productivity benchmarks); (iii) direct hiring cost K; (iv) training time/cost H to full productivity.
- **Morale/fairness** $\phi(\varphi)$: internal equity ranges, pay bands, policy constraints (lower bound of compliant pay).

With these, compute $w_{\min}$ and $w_{\max}$ role-by-role and flag $F^E = (w - w_{\max})_+$.

11 Extensions

11.1 Team production

Let v be defined by a Shapley marginal of the production function to avoid misattribution in teams. Then $w_{\max}$ remains $\bar{w} + \Delta(R)$ and the logic of F^E is unchanged.

11.2 Bargaining

Let wages result from Nash bargaining over surplus $\Pi(w) = v - w$ with fallback replacement loss $-\Delta$. The wage outcome belongs to $[w_{\min}, w_{\max}]$ under standard bargaining weights. Only

$w > w_{\max}$ yields favor.

11.3 Altruistic firms

If the firm objective is $\Pi + \alpha U_{\text{worker}}$ with $\alpha > 0$, then $w > w_{\max}$ can be optimal in that extended objective. Our definition still tags such transfers as "favor" in the *profit* sense; authors may relabel this *philanthropic favor* depending on context.

12 Main Results: Propositions and Proofs

Proposition 8 (Tightness of Upper Bound)**.** *For any (b, φ, R, S), there exists an optimal $w^* \in \mathcal{W}$ that attains the firm's value. If $w > w_{\max}$, replacing (or cutting to $w_{\max}$) strictly increases firm value absent adjustment costs.*

Proof. Within $\mathcal{W}$ the firm is indifferent among wages that satisfy retention and do not exceed replacement value. For $w > w_{\max}$, the definition of $w_{\max}$ implies $v - w < v - \bar{w} + \Delta(R)$, so replacement strictly dominates (or cutting to $w_{\max}$ keeps the match while increasing profit). □

Proposition 9 (Replaceability Reduces Favor Risk)**.** $\partial w_{\max}/\partial R < 0$ *(Proposition 6). Hence for any fixed w, increasing R (better pipelines, faster fills, standardized training) weakly reduces F^E.*

Proposition 10 (Scarcity Raises Favor Risk)**.** $\partial w_{\max}/\partial S > 0$ *and* $\partial w_{\min}/\partial S > 0$*. For fixed w, higher scarcity S (tighter markets) raises both bounds; the probability that $w > w_{\max}$ rises if w does not adjust.*

Proposition 11 (Bounded Favor Under Policy)**.** *If the firm enforces a compensation policy $w \le \bar{w} + \Delta(R) + \epsilon$ with audit tolerance $\epsilon \ge 0$, then $F^E \le \epsilon$ deterministically.*

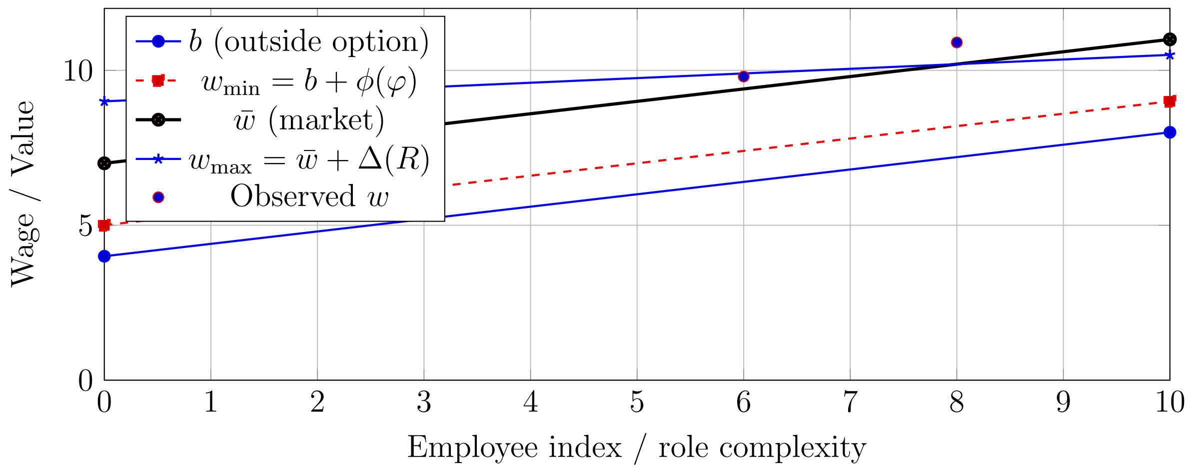
13 Policy and Managerial Implications

1. **Favor screening:** Compute $w_{\max}$ at the role level; flag $w > w_{\max}$ as $F^E > 0$. This neither penalizes efficiency wages nor conflates fair-wage premia with favor.
2. **Reduce F^E by raising R:** invest in recruiting pipelines, standardize onboarding to lower K, H, and reduce ℓ; this tightens the upper bound and shrinks favor risk.
3. **Governance:** institute an audit rule $w \le \bar{w} + \Delta(R) + \epsilon$ with periodic recalibration.

4. **Communication:** present wage bands to managers; encourage *within-band* discretion to maintain morale without creating F^E.

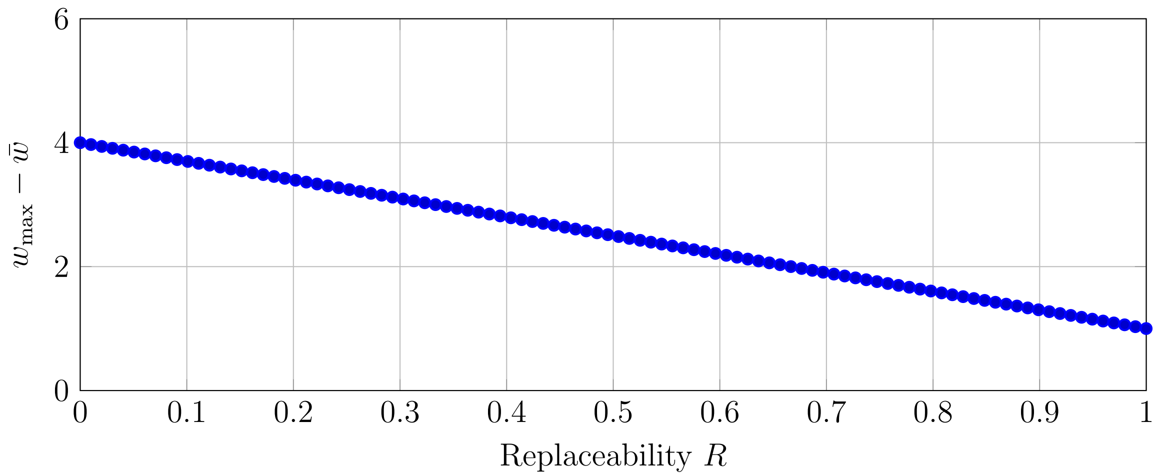
14 Figures (embedded; no external images)

Figure 1: Wage band and Economic Favor



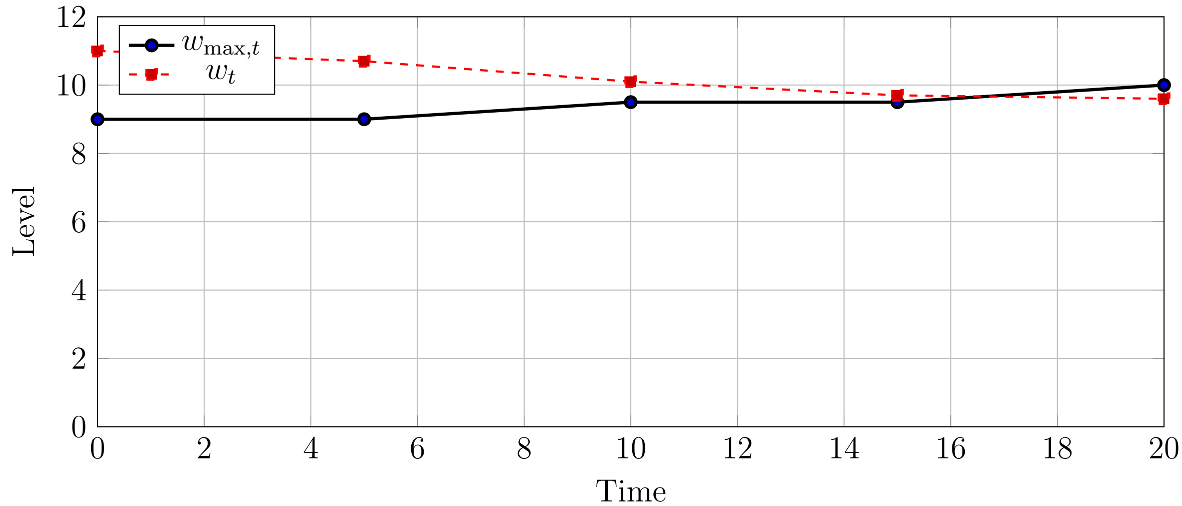
Notes: The value-maximizing wage band is $[w_{\min}, w_{\max}]$. Only observations with $w > w_{\max}$ (e.g., the point at role index ≈ 8) imply $F^E > 0$.

Figure 2: Replaceability tightens the upper bound



Notes: $\Delta(R)$ falls in R, so $w_{\max} - \bar{w}$ declines with replaceability. Favor risk shrinks as R rises.

Figure 3: Dynamic adjustment with convex costs



Notes: With convex adjustment costs, $F_t^E = w_t - w_{\max,t}$ can persist despite being inefficient in a one-shot sense. As costs fall, convergence to the band accelerates.

15 Robustness and Scope Conditions

Our claims are conditional on (i) decomposing observable frictions into $(b, \bar{w}, \Delta, \phi)$, (ii) measuring v net of team spillovers not attributable to others, and (iii) stability of job content. In contexts with volatile v or rapidly changing task mixes, bounds should be recomputed at higher frequency. The theory is silent on normative judgments about philanthropy; it only distinguishes value-consistent premia from discretionary transfers.

16 Conclusion

We developed a disciplined theory of when employment can be described as a "favor." The answer is: only when pay exceeds the *upper bound* of value-consistent wages implied by retention, replacement, training, vacancy delay, and fairness constraints. This reframing resolves the main theoretical weakness of naive definitions and yields clear, testable implications. Our decomposition and measurement plan allow firms and researchers to audit discretionary transfers without penalizing efficient premia.

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Appendix A: Additional Proof Sketches

Lemma 1 (Band Non-emptiness)**.** *If* $b + \phi(\varphi) \leq \bar{w} + \Delta(R)$ *then* $\mathcal{W}$ *is non-empty.*

Proof. Trivial inequality ensures $w_{\min} \leq w_{\max}$. □

Lemma 2 (Team Production Identification). *If v is identified as a Shapley marginal of a symmetric production function with diminishing returns, then v is well-defined and invariant to ordering of workers.*

Appendix B: Implementation Checklist

1. Estimate $\bar{w}$ from postings/surveys; b from realized outside offers; ϕ from equity/policy constraints.
2. Measure ℓ, λ_v, K, H at the job family level to compute $\Delta(R)$.
3. Compute $w_{\min}$, $w_{\max}$ per role; flag $F^E = (w - w_{\max})_+$; report by business unit.

Appendix: Full Proofs and Technical Results

Preliminaries and Notation

Time is discrete, discount factor $\delta \in (0,1)$. A firm matched with worker i at time t chooses a wage w_t (we suppress the i subscript). Let:

- v_t be the worker's period marginal revenue contribution (net of others' spillovers).
- b_t be the outside option; φ_t parametrizes morale/fairness constraints via $\phi(\varphi_t) \geq 0$.
- $R \in [0,1]$ measure replaceability; higher R means shorter vacancy delay $\ell(R)$, lower hiring cost $K(R)$, and lower training cost $H(R)$.
- $\Delta(R) := \ell(R)\lambda_v + K(R) + H(R)$ be the total replacement wedge; assume $\Delta'(R) < 0$.
- $\bar{w}$ denote the steady-state wage for a replacement hire in the same role/market.

We assume stationarity for the baseline results (so we drop t where not needed); dynamics are handled in Section 16.

Bounds and favor. The retention lower bound is

$$w_{\min} \;:=\; b + \phi(\varphi).$$

The replacement upper bound is

$$w_{\max} \;:=\; \bar{w} + \Delta(R).$$

The *value-maximizing wage band* is $\mathcal{W} := [w_{\min}, w_{\max}]$. *Economic Favor* is

$$F^E \; := \; (w - w_{\max})_+ \; = \; \max\{0, \, w - \bar{w} - \Delta(R)\}.$$

The *naïve* wage-productivity gap is $F^A := w - v$.

Lemma A1 (Band Feasibility and Minimal Favor)

The band $\mathcal{W}$ is non-empty iff $w_{\min} \leq w_{\max}$. If $w_{\min} > w_{\max}$, any feasible employment necessarily implies Economic Favor of at least

$$F^E \; \geq \; w_{\min} - w_{\max} \; = \; b + \phi(\varphi) - \bar{w} - \Delta(R).$$

In particular, the smallest feasible wage $w = w_{\min}$ entails $F^E = w_{\min} - w_{\max} > 0$.

Proof. Non-emptiness is equivalent to $w_{\min} \leq w_{\max}$ by definition. If $w_{\min} > w_{\max}$, any feasible w must satisfy $w \geq w_{\min} > w_{\max}$, hence $F^E = (w - w_{\max})_+ \geq w_{\min} - w_{\max}$. The bound is attained at $w = w_{\min}$. □

Proposition A1 (No-Favor Region)

If $w \in [w_{\min}, w_{\max}]$ then $F^E = 0$. In particular, $w > v$ does not by itself imply $F^E > 0$.

Proof. By definition, $F^E = (w - w_{\max})_+$. If $w \leq w_{\max}$, then $F^E = 0$ regardless of v. □

Proposition A2 (Tightness of the Upper Bound)

Fix $(b, \varphi, R, \bar{w})$ and suppose the firm must either retain the incumbent at w or replace. Under stationarity, the one-step value from retention is $V^{ret} = v - w + \delta\mathcal{V}$, while the one-step value from replacement is $V^{rep} = -\Delta(R) + v - \bar{w} + \delta\mathcal{V}$, where $\mathcal{V}$ is the continuation value after the transition. Then:

(a) *Retain iff $w \leq \bar{w} + \Delta(R)$; replace iff $w > \bar{w} + \Delta(R)$.*

(b) *Any $w \in [w_{\min}, w_{\max}]$ is value-consistent; if $w > w_{\max}$, replacement (or cutting to $w_{\max}$ if feasible) strictly increases firm value absent wage adjustment frictions.*

Proof. The difference $V^{\text{ret}} - V^{\text{rep}} = (v - w) - (v - \bar{w} - \Delta(R)) = \bar{w} + \Delta(R) - w$. Part (a) follows. For (b), if $w \leq w_{\max}$ (and $\geq w_{\min}$), retention is weakly optimal. If $w > w_{\max}$, then $V^{\text{rep}} > V^{\text{ret}}$, so replacing strictly increases value. If cutting the wage to $\hat{w} \leq w_{\max}$ is feasible (no frictions), then $v - \hat{w} > v - w$ also strictly increases value. □

Proposition A3 (Comparative Statics of the Band)

Suppose $\Delta'(R) < 0$, $\bar{w}'(S) > 0$, $b'(S) > 0$, and $\phi'(\varphi) \geq 0$. Then

$$\frac{\partial w_{\max}}{\partial R} = \Delta'(R) < 0, \qquad \frac{\partial w_{\max}}{\partial S} = \bar{w}'(S) > 0, \qquad \frac{\partial w_{\min}}{\partial S} = b'(S) > 0, \qquad \frac{\partial w_{\min}}{\partial \varphi} = \phi'(\varphi) \geq 0.$$

Hence, scarcity S shifts the band upward; replaceability R tightens the upper bound downward.

Proof. Differentiate $w_{\max} = \bar{w}(S) + \Delta(R)$ and $w_{\min} = b(S) + \phi(\varphi)$ componentwise. □

Proposition A4 (Replaceability Reduces Favor Risk)

Fix an observed wage w and $\bar{w}, S, \varphi$. If R increases (replacement becomes easier), the upper bound $w_{\max}$ falls and the indicator $\mathbf{1}\{w > w_{\max}\}$ weakly decreases. In particular, F^E weakly decreases in R.

Proof. From Proposition 16, $\partial w_{\max}/\partial R < 0$. For fixed w, the event $\{w > w_{\max}\}$ becomes weakly less likely as R rises, and $F^E = (w - w_{\max})_+$ is a decreasing function of $w_{\max}$. □

Proposition A5 (Scarcity Raises Favor Risk)

Fix w, R, φ. If scarcity S increases, both $w_{\min}$ and $w_{\max}$ increase; the indicator $\mathbf{1}\{w > w_{\max}\}$ weakly increases and F^E weakly increases (unless w adjusts).

Proof. By Proposition 16, $\partial w_{\max}/\partial S > 0$ and $\partial w_{\min}/\partial S > 0$. For fixed w, a higher $w_{\max}$ makes $w > w_{\max}$ weakly more likely, hence F^E weakly rises. □

Proposition A6 (Decomposition Identity)

For any $(w, v, \bar{w}, R, b, \varphi)$,

$$w - v = \underbrace{(\bar{w} - v)}_{\text{market premium}} + \underbrace{\Delta(R)}_{\text{replacement wedge}} + \underbrace{(w - \bar{w} - \Delta(R))_+}_{\text{Economic Favor}} + \underbrace{\phi(\varphi) + (b - \bar{w}) - (w - \bar{w} - \Delta(R))_-}_{\text{retention/fairness misalignment}},$$

where $x_- := \max\{0, -x\}$. In particular, a positive naïve gap $w - v > 0$ does not identify $F^E > 0$.

Proof. Add and subtract $\bar{w} + \Delta(R)$:

$$w - v = (\bar{w} - v) + \Delta(R) + \big(w - \bar{w} - \Delta(R)\big).$$

Decompose the last term into positive and negative parts: $x = x_+ - x_-$. Group terms as indicated. The Economic Favor piece is $x_+ = (w - \bar{w} - \Delta(R))_+$. The remainder bundles the retention/fairness part (and, when $w < \bar{w} + \Delta$, a negative slack $-(\cdot)_-$). $\square$

Proposition A7 (Bounded Favor Under Policy)

Suppose the firm enforces a compensation policy $w \le \bar{w} + \Delta(R) + \epsilon$ with audit tolerance $\epsilon \ge 0$. Then deterministically $F^E \le \epsilon$.

Proof. By definition, $F^E = (w - \bar{w} - \Delta(R))_+ \le \epsilon$. $\square$

Proposition A8 (Band Feasibility, Dependence Asymmetry, and Forced Favor)

(a) The wage band is feasible iff $b + \phi(\varphi) \le \bar{w} + \Delta(R)$. (b) If $b + \phi(\varphi) > \bar{w} + \Delta(R)$, then any feasible wage must satisfy $w \ge w_{\min} > w_{\max}$, so $F^E \ge w_{\min} - w_{\max} > 0$. In this case, employment necessarily embeds favor of at least $b + \phi(\varphi) - \bar{w} - \Delta(R)$.

Proof. Part (a) is Lemma 16. Part (b) follows immediately: the lower bound exceeds the upper bound, so any $w \ge w_{\min}$ implies $w > w_{\max}$ and thus positive F^E of at least the gap. $\square$

Remark A1 (On "Wage Growth vs. Productivity Growth")

The statement "favor persists when $\Delta W \ge \Delta V$" applies to the *naïve* gap $F^A = w - v$, not directly to F^E (which depends on the threshold $\bar{w} + \Delta$). Formally:

Lemma 3 (Naïve Gap Dynamics)**.** *If $w_{t+1} - w_t \ge v_{t+1} - v_t$ then $F^A_{t+1} \ge F^A_t$. If additionally $w_{\max}$ is constant over t and $w_t > w_{\max}$, then $F^E_{t+1} \ge F^E_t$; and if $w_t \le w_{\max} < w_{t+1}$ then $F^E_{t+1} > 0$.*

Proof. $F^A_{t+1} - F^A_t = (w_{t+1} - w_t) - (v_{t+1} - v_t) \ge 0$. For F^E, if $w_{\max}$ is constant, $F^E_t = (w_t - w_{\max})_+$ is increasing in w_t. Thus the claims follow. $\square$

Dynamic Persistence with Adjustment Costs

We now formalize persistence when changing wages is costly. Let $C : \mathbb{R} \to \mathbb{R}_+$ be convex, continuously differentiable, with $C(0) = 0$ and $C'(\cdot)$ increasing. Given sequences $\{w_{\max,t}\}$ (from $\bar{w}(S_t) + \Delta(R_t) - \Gamma_t$), the firm chooses $\{w_t\}_{t \ge 0}$ to minimize the discounted loss

$$\min_{\{w_t\}} \sum_{t=0}^{\infty} \delta^t \Big\{ L_t(w_t) + C(w_t - w_{t-1}) \Big\}, \qquad L_t(w) := (w - w_{\max,t})_+, \tag{10}$$

where w_{-1} is given. This is a convex dynamic control problem.

Proposition 12 (First-Order Conditions and Monotone Adjustment)**.** *Suppose C is strictly convex and $w \mapsto L_t(w)$ is convex (it is). Then an optimal sequence exists and satisfies the Euler conditions*

$$\delta\, C'(w_{t+1} - w_t) - C'(w_t - w_{t-1}) \;\in\; \partial L_t(w_t), \quad t \geq 0,$$

where ∂L_t is the subdifferential of L_t. In particular, if $w_t > w_{\max,t}$ then $\partial L_t(w_t) = \{1\}$; if $w_t < w_{\max,t}$ then $\partial L_t(w_t) = \{0\}$; and if $w_t = w_{\max,t}$ then $\partial L_t(w_t) = [0, 1]$.

Proof. The objective in (10) is a proper, lower semicontinuous, strictly convex functional in $\{w_t\}$ (strict convexity comes from C). Standard results for convex dynamic programs yield existence and the Euler conditions (subgradient optimality) by summing first-order optimality conditions; see, e.g., Rockafellar's convex analysis arguments. The subdifferential of L_t follows from the hinge-loss definition. □

Corollary 2 (Persistence)**.** *If $w_{t-1} \gg w_{\max,t}$ and C' is steep near 0 (high marginal cost of cutting wages), then the optimal policy reduces w_t gradually and $F_t^E = w_t - w_{\max,t}$ remains positive for multiple periods. If $C \equiv 0$, the unique optimizer is $w_t = \min\{w_{t-1}, w_{\max,t}\}$, so F_t^E is eliminated in one step whenever $w_{t-1} > w_{\max,t}$.*

Proof. With C' steep near zero, the Euler condition forces small (not discrete) changes in $w_t - w_{t-1}$ to balance the unit subgradient $\partial L_t(w_t) = \{1\}$ when $w_t > w_{\max,t}$, implying gradual descent. When $C \equiv 0$, the period problem is separable and trivially solved by projecting w onto $(-\infty, w_{\max,t}]$ each period. □

Discussion: Asymmetric Dependence

Heuristically, "employee depends more on the firm" corresponds to low b and/or small $\phi(\varphi)$, while "firm depends more on the employee" corresponds to large $\Delta(R)$ (low R). Proposition 16 shows that *favor is forced* precisely when the retention lower bound exceeds the replacement upper bound ($w_{\min} > w_{\max}$). In contrast, if b is low and $\Delta(R)$ is small (employee depends more, firm depends less), then $w_{\min}$ is small and $w_{\max}$ is small; favor is *not* mechanically implied—indeed, F^E is less likely. This pins down the exact logic conditions under which the intuitive "favor" claim is theoretically valid in our framework.

Q.E.D.