When Does Government Debt Increase the Money Supply? A Framework for Monetization and Inflation

March 2025[a,1]

[a] *economics*
[b] *business*

Abstract

This paper develops a theoretical framework to analyze the inflationary consequences of government debt monetization. We introduce the concept of a dynamic monetization share, θ_t, defined as the fraction of new government debt that is effectively financed through increases in the money supply. Embedding this construct into a modified version of the quantity theory of money, we derive several testable propositions linking θ_t to inflation outcomes. The model shows that inflation is increasing in θ_t, with potentially nonlinear effects when monetization exceeds critical thresholds. This framework provides a tractable link between fiscal deficits, monetary accommodation, and inflation dynamics, particularly in high-debt or low-growth environments. While theoretical in nature, our approach offers empirically implementable tools for assessing inflation risk and helps explain recent inflation patterns that traditional models struggle to capture, most notably, the muted inflation following the 2008 crisis versus the surge after the COVID-19 fiscal response.

Keywords: Debt monetization, Inflation dynamics, Fiscal-monetary interaction, Quantity theory of money, Inflation risk, Monetary accommodation, High-debt environments

JEL Classification: E31, E52, E62, H63, E58

1. Introduction

Recent episodes of rising inflation, especially in the aftermath of large-scale fiscal responses to economic crises, have reignited interest in the relationship between government debt and inflation. Traditional monetarist views emphasize money growth as the proximate driver of inflation Friedman (1968); Lucas (1980), while modern fiscal theories emphasize the role of fiscal backing and debt sustainability Leeper (1991); Woodford (2001).

Despite these theoretical advances, a central empirical challenge remains: identifying the specific mechanisms through which government borrowing translates into money creation and, ultimately, inflation. In particular, the interaction between fiscal deficits and central bank balance sheet expansions (often labeled "monetization") has become increasingly relevant in light of recent policy episodes (Sargent and Wallace, 1981; Blanchard, 2019).

In the aftermath of the COVID-19 pandemic, the United States undertook one of the largest peacetime fiscal expansions in its history. Government debt surged from 79 percent of GDP in 2019 to over 100 percent by 2022, fueled by emergency spending, income transfers, and automatic stabilizers (References). At the same time, the Federal Reserve expanded its

*Corresponding author

balance sheet by over 4 trillion USD, reaching nearly 9 trillion USD at the start of 2022, in part through large-scale purchases of Treasury securities (References). This confluence of fiscal expansion and monetary accommodation has renewed intense debate over the relationship between government debt, money creation, and inflation — particularly as US inflation reached multi-decade highs in 2021–2022 (References). Despite this urgency, there remains a lack of formal frameworks to quantify the degree to which rising government debt translates into money supply growth — and, by extension, inflation (References).

This paper introduces a tractable framework to address this gap. We define a key state variable — the monetization share θ_t — as the fraction of newly issued government debt that is effectively financed by increases in the money supply. This variable captures the degree of monetary accommodation associated with fiscal policy, encompassing not only direct central bank purchases of debt but also indirect channels such as commercial bank purchases that lead to deposit creation. In doing so, our framework extends the traditional quantity theory of money by embedding fiscal dynamics directly into the money supply process.

We show that inflation is increasing in both the rise in government debt and the monetization share θ_t, and that this relationship may be nonlinear — particularly when θ_t crosses critical thresholds that alter expectations or induce shifts in money velocity. Importantly, even in the absence of overt monetary financing, financial intermediation can give rise to effective monetization when banks expand deposit liabilities to absorb public debt. This suggests that large fiscal expansions may be more inflationary than conventional models predict, especially if it is financed with large amounts of money increases, prevailing inflation is high and the rise in money occurs from a relatively low base, i.e. when the prevailing base of money stock is low, especially when conducted in an environment of ample reserves and constrained output productivity growth — features that closely describe the U.S. macroeconomic landscape in the post-pandemic period.

By making θ_t explicit and time-varying, our framework allows for a more granular assessment of how fiscal and monetary authorities interact to influence inflation outcomes. We derive testable propositions that link debt monetization share to inflation, controlling for velocity and output growth, and show that inflation may respond more aggressively to monetization in low-growth environments. This may help explain why inflation remained muted in earlier episodes of high debt (e.g., post-2008), but surged following the pandemic — when both the size and the monetization of debt reached unprecedented levels, prevailing inflation is high, and the stock of money starting from a relatively low base and accelerated quickly.

Our paper contributes to several strands of literature. First, it provides a mechanism within the monetary-fiscal policy nexus, complementing the fiscal theory of the price level by focusing on the intermediate role of money supply dynamics via the monetization share. Second, it operationalizes the classic insight that "inflation is always and everywhere a monetary phenomenon," but in a way that endogenizes monetary expansion through fiscal actions. Third, it speaks directly to ongoing policy debates in the U.S. about the inflationary consequences of sustained deficits, debt sustainability, and the appropriate degree of central bank accommodation.

The remainder of the paper proceeds as follows. Section 2 presents a brief literature reviews. Section 3 details the theoretical model and its main propositions. Section 4 discusses policy implications for US fiscal and monetary authorities. Section 5 concludes and suggests avenues for further research.

2. Literature Review and Contribution

The relationship between government debt, money creation, and inflation has been the subject of longstanding debate in macroeconomics. Classical monetarist frameworks (Friedman, 1968; Sargent and Wallace, 1981) emphasize the primacy of money growth in driving inflation, often treating fiscal variables as exogenous or subordinate to monetary rules. In contrast, the fiscal theory of the price level (FTPL) (Leeper, 1991; Sims, 1994; Woodford, 2001) places fiscal policy at the center of inflation determination, arguing that the price level adjusts to ensure government solvency when monetary policy is passive. While these perspectives differ in causal ordering, both acknowledge that inflation reflects the interaction of fiscal and monetary stances — yet neither fully formalizes the dynamic process by which government debt issuance is converted into money.

More recently, empirical and theoretical studies have examined the inflationary consequences of large-scale asset purchases (Gagnon et al., 2011; Greenwood et al., 2015), central bank balance sheet policies (Del Negro and Sims, 2015; Brunnermeier and Sannikov, 2016), and unconventional monetary–fiscal coordination (Farhi and Werning, 2017; Bianchi and Melosi, 2019). These papers show that central bank accommodation of fiscal expansions — especially in low interest rate environments — can have nontrivial inflationary effects. However, most do not explicitly model the fraction of debt monetized over time or provide a metric to measure it empirically. Moreover, they often treat the central bank as the sole agent capable of creating money, underemphasizing the role of commercial banks and financial intermediaries in endogenously expanding the money supply in response to fiscal issuance.

Our work contributes to this literature in three key ways. First, in via a theoretical framework where we introduce the concept of the monetization share, θ_t, as a continuous, time-varying parameter capturing the share of new government debt that is financed through money creation. Unlike the binary treatment of monetization in earlier models (e.g., "money financing" vs. "bond financing"), our framework allows for a spectrum of outcomes depending on the actions of central banks, commercial banks, and financial market participants. Second, via unifying monetarist and fiscal channels by embedding θ_t in a modified version of the quantity theory of money, we bridge the monetarist emphasis on money growth with the FTPL emphasis on fiscal solvency, showing how fiscal expansions that are partially monetized can drive inflation — even in the presence of substantial output gaps. This unified framework helps explain empirical puzzles such as why inflation was muted after the 2008 financial crisis but surged following the COVID-19 response, despite similar levels of fiscal stimulus. Third, via operationalization for empirical work where we provide a practical method for constructing θ_t using observable U.S. data from the Federal Reserve's H.4.1 reports and the Treasury's Monthly Statement of the Public Debt. To our knowledge, this is the first paper to propose and quantify a time-varying debt monetization share for empirical inflation modeling. This enables new empirical strategies to test for threshold effects, nonlinearities, and expectational shifts in the inflation process that depend on the degree of debt monetization.

Our work is also related to recent studies on inflation forecasting and fiscal dominance (Cochrane, 2023; Hall and Sargent, 2022), which argue that standard models understate the inflationary consequences of large fiscal shocks. However, while these studies focus on long-run solvency conditions or regime shifts, our framework targets a specific and measurable short-run channel — money creation via fiscal issuance — and demonstrates how this channel amplifies inflation depending on prevailing macroeconomic conditions.

In this sense, our paper contributes a novel identification strategy to an active debate in both policy and academia. It offers a tractable and empirically implementable tool to measure the real-time inflationary risk of fiscal expansions, and sheds light on the mechanics of money creation in high-debt environments — an area that, despite its importance, has been underexplored in the modern literature.

3. Theoretical Framework

We assume the government runs a budget deficit and has debt service obligation, both of which it finances with new debt issuance. Suppose that government issues new debt ΔD_t to finance its deficit and debt service, then, assuming no borrowing beyond financing deficit and debt service, then this new debt must finance the deficit and debt service:

$$\Delta D_t = G_t - T_t + i_{t-1}B_{t-1} \tag{1}$$

where G_t is government spending, T_t is tax revenue and $i_{t-1}B_{t-1}$ is interest on the previous debt stock.

Generally, the new government debt can be absorbed solely, partly or jointly by three broad entities. First, it can be absorbed by the central bank. Second, it can be absorbed by commercial banks and third it can be absorbed by the investing public comprising household or firms and other non-depositing taking institutions. And depending on the entity which absorbes the government debt, money supply may need to grow to be able to purchase this new debt, meaning there could exist a time varying share of the new government debt that is financed with new money, that's a rise in money supply. This means that a share of this new debt could be monetized.

Thus, define θ_t as the share of ΔD_t monetized, then it must be true that:

$$\Delta M_t = \theta_t \Delta D_t \tag{2}$$

When the monetization share $\theta_t = 1$, the entire government deficit and debt service — given by $G_t - T_t + i_{t-1}B_{t-1}$ and financed through new debt issuance ΔD_t — is matched by an equivalent increase in the money supply, i.e., $\Delta M_t = \Delta D_t$. This implies full monetization: either the central bank prints new base money or commercial banks create deposits ex nihilo to absorb the newly issued debt. Conversely, when $\theta_t = 0$, there is no change in the money supply ($\Delta M_t = 0$), so the government's borrowing does not lead to monetary expansion. In this case, the new debt is absorbed entirely through existing financial resources — commercial banks may reallocate reserves or deposits, and non-bank entities (households and firms) may use existing savings to purchase government bonds. No new deposits or base money are created, and as a result, the money supply remains unchanged or may even contract.

More generally, when θ_t is increasing, a growing share of new debt is financed through monetary expansion, implying that the larger the deficit, the greater the associated increase in the money supply. This occurs either via central bank asset purchases or commercial banks creating new deposits as they acquire government securities, often indirectly through primary dealers. As θ_t rises, the inflationary impact of deficits becomes more pronounced — particularly if output does not expand commensurately — due to the excess liquidity injected into the economy. On the other hand, if θ_t is decreasing, a smaller fraction of the deficit is monetized, meaning that fiscal expansions are increasingly funded through real resource transfers from the private sector. In such cases, the inflationary consequences are dampened, and the macroeconomic burden shifts toward higher interest rates or potential crowding out of private investment.

Finally, from the quantity theory of money, we know that:

$$P_t Y_t = M_t V_t \tag{3}$$

which, after computing the approximate product changes, rewrites more precisely as

$$\frac{\pi_t}{1+\pi_t} + \frac{y_t^g}{1+y_t^g} = \frac{\theta_t \Delta D_t}{M_t} + \frac{v_t^g}{1+v_t^g} \tag{4}$$

where:

- $\pi_t = \frac{\Delta P_t}{P_{t-1}}$ is the inflation rate, i.e., the growth rate of the price level,
- $y_t^g = \frac{\Delta Y_t}{Y_{t-1}}$ is the real output (GDP) growth rate,
- $v_t^g = \frac{\Delta V_t}{V_{t-1}}$ is the growth rate of money velocity,
- θ_t is the monetization share — the proportion of newly issued government debt that is financed by an increase in the money supply,
- ΔD_t denotes the flow of new government debt,
- M_t is the nominal money stock.

This is the differential form of the quantity theory of money, modified to reflect growth dynamics in inflation, output, and money velocity. This equation expresses the identity that growth in nominal aggregate demand (inflation plus real output growth) must be matched by growth in effective monetary resources, either through a monetized fiscal expansion or an increase in velocity.

The term $\frac{\theta_t \Delta D_t}{M_t}$ captures the inflationary impulse arising from monetized deficit financing. As θ_t increases — indicating a larger fraction of debt is financed by money creation — the contribution of fiscal actions to inflation grows. Meanwhile, $\frac{v_t^g}{1+v_t^g}$ accounts for changes in the circulation speed of money, which can amplify or dampen the inflationary effect of fiscal policy.

In low-growth or stagnant output environments (i.e., when y_t^g is small), the inflationary effect of θ_t becomes more pronounced. Thus, this framework offers a tractable link between fiscal dynamics, money creation, and inflation — especially relevant in high-debt economies like the United States, where post-crisis policy responses have intensified the interaction between fiscal and monetary institutions. This leads to the following propositions.

Proposition 1: If θ_t is increasing and ΔD_t, M_t are positive, then inflation π_t increases with θ_t, that is:

$$\frac{\partial \pi_t}{\partial \theta_t} = \frac{\Delta D_t}{M_t} > 0 \tag{5}$$

Proposition 2: The marginal effect of debt monetization (θ_t) on inflation (π_t) is decreasing in the growth rate of real output (y_t^g) and increasing in the growth rate of money velocity (v_t^g).

Given the inflation equation:

$$\frac{\pi_t}{1+\pi_t} = \frac{\theta_t \Delta D_t}{M_t} + \frac{v_t^g}{1+v_t^g} - \frac{y_t^g}{1+y_t^g} \tag{6}$$

it follows that:

$$\frac{\partial^2 \pi_t}{\partial \theta_t \partial y_t^g} < 0, \quad \text{and} \quad \frac{\partial^2 \pi_t}{\partial \theta_t \partial v_t^g} > 0. \tag{7}$$

When real output growth (y_t^g) is high, the economy is more capable of absorbing the increases in the money supply created to purchase government debt, without generating inflation. Conversely, when money velocity growth (v_t^g) is high, inflationary pressure of monetized debt increases, as each dollar circulates more rapidly in the economy.

4. Policy Implications and Extensions

- Central banks must monitor θ_t alongside inflation forecasts.
- Fiscal authorities should consider the inflationary costs of monetizing debt.
- Extensions can incorporate open economy effects (e.g., exchange rate pass-through), financial frictions, and adaptive expectations.
- Central banks should monitor θ_t as a leading indicator of inflationary pressure, especially in fiscal expansions.
- Fiscal authorities need to assess the inflationary risk of monetization strategies and consider their interaction with private bank behavior.
- Macroprudential tools should be calibrated to mitigate risks from endogenous money creation via bank financing of government deficits.

5. Conclusion

This paper introduces a tractable framework to study how government debt monetization affects inflation. By explicitly modeling the monetization share θ_t, we link fiscal operations to changes in the money supply and inflation dynamics. The framework yields clear testable implications for policy, especially in high-debt or low-output environments. Future work could extend the analysis to include exchange rate effects, inflation expectations, and central bank credibility.

This framework offers a tractable and empirically grounded approach to understanding how debt monetization drives inflation. By explicitly modeling θ_t, we provide new theoretical insights and empirical tools for policymakers to assess the inflationary consequences of fiscal expansions, particularly in the presence of central bank accommodation.

First, we introduce the concept of the monetization share, θ_t, as a continuous, time-varying parameter that captures the fraction of new government debt financed through money creation rather than bond issuance. This generalizes earlier binary frameworks that classify deficit financing as either entirely "money-financed" or "bond-financed." By allowing θ_t to vary between zero and one, our model reflects the nuanced and evolving behavior of central banks, commercial banks, and financial markets in absorbing government debt.

Second, we embed θ_t into a modified quantity theory of money that unifies traditional monetarist and fiscal perspectives. This framework bridges the monetarist emphasis on money growth with the fiscal theory of the price level (FTPL)'s focus on government solvency. The result is a tractable mechanism in which partially monetized fiscal expansions can generate inflation — even when real economic activity remains depressed.

This perspective helps resolve a core empirical puzzle: why similar levels of fiscal stimulus following the 2008 financial crisis and the COVID-19 pandemic produced markedly different inflation outcomes. While both episodes featured historically large deficits, inflation remained subdued post-2008 but surged post-COVID. Our framework attributes this divergence to variation in θ_t across the two episodes: a lower monetization share in the earlier period muted inflationary pressures, while a higher share in the later period amplified them. This suggests that the inflationary consequences of fiscal expansions depend critically not just on their size but on their mode of financing.

Inflation didn't surge post-2008 but surged post-COVID despite large deficits. However, just a few offer a tractable model that directly links that divergence to the degree of monetization. Our model shows that It's not just how much debt is issued — it's how it's financed that drives inflation.

References

Bianchi, F. and L. Melosi (2019). The Dire Effects of the Lack of Monetary and Fiscal Coordination. *Journal of Monetary Economics 104*, 1–22.

Blanchard, O. (2019). Public Debt and Low Interest Rates. *American Economic Review 109*(4), 1197–1229.

Brunnermeier, M. K. and Y. Sannikov (2016). The I Theory of Money. Working Paper 22533, National Bureau of Economic Research.

Cochrane, J. H. (2023). *The Fiscal Theory of the Price Level*. Princeton University Press.

Del Negro, M. and C. A. Sims (2015). When Does a Central Bank's Balance Sheet Require Fiscal Support? *Journal of Monetary Economics 73*, 1–19.

Farhi, E. and I. Werning (2017). Fiscal Multipliers: Liquidity Traps and Currency Unions. In J. B. Taylor and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Volume 2B, pp. 2417–2492. Elsevier.

Friedman, M. (1968). The Role of Monetary Policy. *American Economic Review 58*(1), 1–17.

Gagnon, J., M. Raskin, J. Remache, and B. Sack (2011). The Financial Market Effects of the Federal Reserve's Large-Scale Asset Purchases. *International Journal of Central Banking 7*(1), 3–43.

Greenwood, R., S. Hanson, and J. C. Stein (2015). A Comparative-Advantage Approach to Government Debt Maturity. *Journal of Finance 70*(4), 1683–1722.

Hall, G. J. and T. J. Sargent (2022). Three World Wars: Fiscal-Monetary Consequences. *American Economic Journal: Macroeconomics 14*(3), 105–142.

Leeper, E. M. (1991). Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies. *Journal of Monetary Economics 27*(1), 129–147.

Lucas, R. E. (1980). Two Illustrations of the Quantity Theory of Money. *The American Economic Review 70*(5), 1005–1014.

Sargent, T. J. and N. Wallace (1981). Some Unpleasant Monetarist Arithmetic. *Federal Reserve Bank of Minneapolis Quarterly Review 5*(3), 1–17.

Sims, C. A. (1994). A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy. *Economic Theory 4*(3), 381–399.

Woodford, M. (2001). Fiscal Requirements for Price Stability. *Journal of Money, Credit and Banking 33*(3), 669–728.

A. Proofs of the Propositions

A.1. Proof of Proposition 1

If θ_t increases and $\Delta D_t > 0$, $M_t > 0$, then inflation π_t increases with θ_t, that is:

$$\frac{\partial \pi_t}{\partial \theta_t} > 0$$

Proof. We know from the model that:

$$\frac{\pi_t}{1+\pi_t} + \frac{y_t^g}{1+y_t^g} = \frac{\theta_t \Delta D_t}{M_t} + \frac{v_t^g}{1+v_t^g} \tag{A.1}$$

Rewriting, we isolate the inflation component:

$$\frac{\pi_t}{1+\pi_t} = \frac{\theta_t \Delta D_t}{M_t} + \frac{v_t^g}{1+v_t^g} - \frac{y_t^g}{1+y_t^g} \tag{A.2}$$

Define the right-hand side as S_t:

$$S_t := \frac{\theta_t \Delta D_t}{M_t} + \frac{v_t^g}{1+v_t^g} - \frac{y_t^g}{1+y_t^g} \tag{A.3}$$

Then, $\frac{\pi_t}{1+\pi_t} = S_t \Rightarrow \pi_t = \frac{S_t}{1-S_t}$, for $S_t < 1$. Taking the partial derivative of π_t wrt θ_t:

$$\frac{\partial \pi_t}{\partial \theta_t} = \frac{\partial}{\partial \theta_t}\left(\frac{S_t}{1-S_t}\right) = \frac{1}{(1-S_t)^2} \cdot \frac{\partial S_t}{\partial \theta_t} \tag{A.4}$$

Compute the partial derivative of S_t with respect to θ_t:

$$\frac{\partial S_t}{\partial \theta_t} = \frac{\partial}{\partial \theta_t}\left(\frac{\theta_t \Delta D_t}{M_t}\right) = \frac{\Delta D_t}{M_t} \tag{A.5}$$

Therefore,

$$\frac{\partial \pi_t}{\partial \theta_t} = \frac{\Delta D_t}{M_t} \cdot \frac{1}{(1-S_t)^2} \tag{A.6}$$

Note that from earlier:

$$S_t = \frac{\pi_t}{1+\pi_t} \quad \Rightarrow \quad 1 - S_t = \frac{1}{1+\pi_t}, \quad \Rightarrow \quad (1-S_t)^2 = \left(\frac{1}{1+\pi_t}\right)^2 \tag{A.7}$$

Substitute into the derivative expression:

$$\frac{\partial \pi_t}{\partial \theta_t} = \frac{\Delta D_t}{M_t} \cdot (1+\pi_t)^2 \tag{A.8}$$

Since $\Delta D_t > 0$, $M_t > 0$, and $(1+\pi_t)^2 > 0$ for $\pi_t > -1$, it follows that: $\frac{\partial \pi_t}{\partial \theta_t} > 0$. □

A.2. Proof of Proposition 2

Proof. Recall that inflation satisfies

$$\pi_t = \frac{X}{1-X}, \quad \text{where} \quad X := \theta_t \frac{\Delta D_t}{M_t} + \frac{v_t^g}{1+v_t^g} - \frac{y_t^g}{1+y_t^g} \tag{A.9}$$

Expressing X in terms of π_t,

$$X = \frac{\pi_t}{1+\pi_t} \implies (1-X) = \frac{1}{1+\pi_t} \tag{A.10}$$

Taking the partial derivative of inflation with respect to the monetization share θ_t, we get

$$\frac{\partial \pi_t}{\partial \theta_t} = \frac{1}{(1-X)^2} \cdot \frac{\Delta D_t}{M_t} = (1+\pi_t)^2 \frac{\Delta D_t}{M_t} > 0 \tag{A.11}$$

Differentiating this expression with respect to output growth y_t^g, yields

$$\frac{\partial^2 \pi_t}{\partial \theta_t \partial y_t^g} = \frac{\partial}{\partial y_t^g}\left((1+\pi_t)^2 \frac{\Delta D_t}{M_t}\right) = 2(1+\pi_t)\frac{\partial \pi_t}{\partial y_t^g} \cdot \frac{\Delta D_t}{M_t} \tag{A.12}$$

Using

$$\frac{\partial \pi_t}{\partial y_t^g} = \frac{\partial}{\partial y_t^g}\left(\frac{X}{1-X}\right) = \frac{1}{(1-X)^2} \cdot \frac{\partial X}{\partial y_t^g} = (1+\pi_t)^2 \cdot \left(-\frac{1}{(1+y_t^g)^2}\right) < 0 \tag{A.13}$$

$$\frac{\partial^2 \pi_t}{\partial \theta_t \partial y_t^g} = -2\frac{\Delta D_t}{M_t}(1+\pi_t)^3 \frac{1}{(1+y_t^g)^2} < 0 \tag{A.14}$$

Similarly, differentiating with respect to money velocity growth v_t^g, we find

$$\frac{\partial^2 \pi_t}{\partial \theta_t \partial v_t^g} = 2\frac{\Delta D_t}{M_t}(1+\pi_t)^3 \frac{1}{(1+v_t^g)^2} > 0 \tag{A.15}$$

□

Remark. Inflation and its drivers satisfy the following partial differential equation:

$$(1+v_t^g)^2 \frac{\partial^2 \pi_t}{\partial \theta_t \partial y_t^g} + (1+y_t^g)^2 \frac{\partial^2 \pi_t}{\partial \theta_t \partial v_t^g} = 0, \tag{A.16}$$

highlighting the intrinsic balance between the inflation sensitivities to output and velocity growth under variations in the monetization share θ_t.

A.3. Proposition 7 (Inflation Threshold Nonlinearity)

Statement: There exists a threshold value $\bar{\theta}$ such that:

$$\frac{d^2 \pi_t}{d\theta_t^2} > 0 \quad \text{for} \quad \theta_t > \bar{\theta}$$

That is, the inflationary impact of monetization becomes increasingly large as the share of debt monetized exceeds a critical level, due to convex (nonlinear) responses from expectations or velocity amplification.

Proof. Start with the modified quantity theory equation:

$$\frac{\pi_t}{1+\pi_t} + \frac{y_t^g}{1+y_t^g} = \frac{\theta_t \Delta D_t}{M_t} + \frac{v_t^g}{1+v_t^g} \tag{A.17}$$

Solving for the inflation term:

$$\frac{\pi_t}{1+\pi_t} = \frac{\theta_t \Delta D_t}{M_t} + \frac{v_t^g}{1+v_t^g} - \frac{y_t^g}{1+y_t^g} \tag{A.18}$$

Define the right-hand side as a function $A(\theta_t)$:

$$A(\theta_t) := \frac{\theta_t \Delta D_t}{M_t} + (\text{constants}) \quad \Rightarrow \quad \frac{\pi_t}{1+\pi_t} = A(\theta_t) \tag{A.19}$$

Solving for π_t:

$$\pi_t(\theta_t) = \frac{A(\theta_t)}{1 - A(\theta_t)} \tag{A.20}$$

First derivative:

$$\frac{d\pi_t}{d\theta_t} = \frac{d}{d\theta_t}\left(\frac{A(\theta_t)}{1-A(\theta_t)}\right) = \frac{A'(\theta_t)}{(1-A(\theta_t))^2} \tag{A.21}$$

Since $A'(\theta_t) = \dfrac{\Delta D_t}{M_t} > 0$, we have: $\dfrac{d\pi_t}{d\theta_t} > 0$. The Second derivative:

$$\begin{aligned} \frac{d^2\pi_t}{d\theta_t^2} &= \frac{d}{d\theta_t}\left(\frac{A'(\theta_t)}{(1-A(\theta_t))^2}\right) = \frac{\Delta D_t}{M_t} \cdot \frac{d}{d\theta_t}\left((1-A(\theta_t))^{-2}\right) \\ &= \frac{\Delta D_t}{M_t} \cdot 2 \cdot \frac{\Delta D_t}{M_t} \cdot \frac{1}{(1-A(\theta_t))^3} = 2\left(\frac{\Delta D_t}{M_t}\right)^2 \cdot \frac{1}{(1-A(\theta_t))^3} \end{aligned}$$

As θ_t increases, $A(\theta_t)$ approaches 1, and the denominator $(1-A(\theta_t))^3$ becomes small, making the second derivative large. Therefore: $\frac{d^2\pi_t}{d\theta_t^2} > 0$ for large θ_t. This establishes the convexity of inflation with respect to monetization share and confirms the existence of a threshold $\bar{\theta}$ beyond which the inflationary impact accelerates.

□