A long run Money-Fiscal Structural Model of Exchange Rates with Cross-Country Evidence

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1 Abstract

In the paper, we develop a structural model of exchange rate dynamics that emphasizes the interplay between long-run monetary and fiscal fundamentals. The model extends the traditional flexible-price monetary framework by incorporating fiscal sustainability indicators—specifically, government debt-to-GDP ratios—into the long-run exchange rate equation. Central to our framework is the notion that persistent fiscal imbalances can generate currency misalignments beyond what standard monetary variables can explain. By relaxing uncovered interest parity, our model allows interest rate differentials to capture risk premiums and policy credibility. Using data from six advanced economies (with the U.S. as the numeraire), we provide empirical support for the model's predictions. Countries with higher debt levels relative to the U.S. tend to experience more pronounced currency depreciation. Moreover, we show that incorporating fiscal fundamentals improves both the explanatory power of long-run cointegration relationships and the out-of-sample forecasting performance of structural exchange rate models. Our findings highlight the importance of jointly considering monetary conditions and fiscal credibility in understanding the dynamics of long-run exchange rate behavior.

2 Introduction

It is undeniable that understanding the long-run determinants of nominal exchange rates remains an essential question in international macroeconomics. Early theoretical work, such as the flexible price monetary model, provides a clear link between exchange rates and monetary fundamentals, including the money supply and real output [Poole, 1970, Gandolfo, 1971].

However, empirical tests typically find that exchange rate movements are poorly forecasted by pure monetary models over relatively short horizons: Meese and Rogoff (1983) demonstrate that a naive random-walk specification often outperforms structural monetary models when using quarterly data for OECD countries [Meese and Rogoff, 1983]. Subsequent studies have also documented similar problems, suggesting that omitted non-monetary variables may play a crucial role in exchange-rate determination.

In response to these obstacles, more recent research has sought to improve the monetary model by exploring longer historical samples or panel-cointegration techniques. Rapach and Wohar (2002), for example, show that when using more than a century of annual data spanning the gold standard, Bretton Woods, and modern floating regimes, there is more robust evidence for a stable long-term relationship between exchange rates and monetary fundamentals [Rapach and Wohar, 2002]. Similarly, the research by Sarno and Taylor (2002) finds that incorporating additional information such as terms-of-trade and commodity prices can improve the accuracy of prediction in certain emerging markets. Despite these evidences, many of these long-run frameworks continue to overlook the fiscal stance of governments, even though fiscal shocks can generate persistent real and nominal effects that monetary variables alone cannot capture [Obstfeld and Rogoff, 1995, Eichengreen, 1994].

Subsequently, a growing body of literature underscores the importance of fiscal fundamentals in exchange-rate dynamics. Chinn and Meredith (2004) document that incorporating deficit-to-GDP ratios enhances long-term cointegrating relationships in samples from industrialized economies [Chinn and Meredith, 2004]. Devereux and Lane (2003) developed a theoretical model in which government debt influences exchange rates through its impact on domestic and foreign money demands [Devereux and Lane, 2003]. Empirical studies by Dell'Ariccia and Rabanal (2007) and Chinn (2006) further confirm that fiscal variables, such as government consumption and public debt, can significantly affect real exchange rates over multi-decadal horizons [Dell'Ariccia and Rabanal, 2007, Chinn, 2006].

This paper proposes a flexible price money-fiscal structural model of exchange rate determination, explicitly incorporating government fiscal variables - such as debt to GDP and budget balances - along with traditional monetary fundamentals. By extending the standard monetary model [Poole, 1970, Gandolfo, 1971] to include fiscal impulses from the outset, we account for the possibility that persistent fiscal imbalances generate long-run misalignments that monetary variables alone cannot explain. We then apply this extended framework to a set of emerging and advanced economies. Our objectives are to determine whether adding fiscal variables yields stable long-run cointegrating relationships when pure monetary models fail, and to test if fiscal fundamentals improve out-of-sample exchange-rate forecasts versus both monetary-only models and a random-walk benchmark.

2.1 Comparison to Related Literature

The long-run determinants of exchange rates have traditionally been explored through the flexible-price monetary model pioneered by Poole (1970) and Gandolfo (1971), emphasizing the role of monetary fundamentals such as money supply, interest rates (unless UIRP is assumed), and output differentials. However, as demonstrated by Meese and Rogoff (1983), monetary fundamentals alone often fail to outperform simple benchmarks like the random walk in out-of-sample forecasts at short horizons, highlighting the pure monetary model's limitations and the likely importance of other variables, for example, fiscal variables, to augment existing models in their structural complexities. To address these limitations, several studies have extended the empirical and theoretical frameworks surrounding exchange rate determination. Rapach and Wohar (2002) use long historical data to provide more evidence supporting stable cointegrating relationships between exchange rates and monetary fundamentals, thereby reinforcing the core monetary model for longer-run horizons. Sarno and Taylor (2002) further suggest that incorporating additional variables such as terms-of-trade and commodity prices improves model fit in specific contexts, especially for emerging markets. Nonetheless, a critical omission in much of this literature remains the incorporation of the role of fiscal fundamentals in exchange rate dynamics. Early monetary models largely ignore fiscal variables, despite theoretical and empirical evidence that fiscal imbalances that are funded via new debt are often correlated with money supply and can produce some exchange rate effects that monetary variables alone do not capture. Obstfeld and Rogoff (1995) and Eichengreen (1994) provide foundational insights on the macroeconomic consequences of fiscal imbalances on exchange rates, emphasizing the interaction between fiscal policy and currency values during episodes of debt accumulation or fiscal stress. More recent work explicitly integrating fiscal variables includes Chinn and Meredith (2004), who show that deficit-to-GDP ratios enhance long-run cointegrating relationships in industrialized economies, while Devereux and Lane (2003) develop a theoretical model linking government debt to exchange rate volatility through money demand effects. Empirical studies by Dell'Ariccia and Rabanal (2007) and Chinn (2006) confirm the significance of fiscal variables such as government consumption and public debt in explaining real exchange rate movements over long horizons. Despite these advances, existing models tend to incorporate fiscal variables only in an ad hoc or secondary fashion, often maintaining the traditional uncovered interest parity (UIP) condition and omitting key distortions related to risk premia and policy credibility that fiscal imbalances generate. Moreover, there has been limited systematic empirical work combining fiscal and monetary fundamentals in a unified structural framework that explicitly allows the interest rate differential to reflect deviations from UIP as well as incorporates fiscal conditions within a monetary model. This paper contributes to the literature by developing a long-run structural money-fiscal model of exchange rate determination that explicitly incorporates government fiscal debt within a flexible-price monetary framework. Unlike prior models, this approach relaxes the standard UIP assumption, allowing the interest rate differential to capture risk premia and credibility effects associated with fiscal sustainability. This innovation addresses an important empirical shortcoming by modeling persistent fiscal imbalances as drivers of currency misalignments beyond the explanatory scope of pure monetary variables. Incorporating fiscal variables improves nominal exchange rate forecasts, outperforming the random-walk benchmark that has been known to rival the pure monetary-only structural models in forecasting capacity. This result highlights the practical forecasting value of fiscal considerations in exchange rate modeling, a dimension often overlooked in earlier work. By integrating monetary and fiscal fundamentals within a coherent theoretical and empirical framework, this paper advances exchange rate modeling in several ways. First, it extends traditional monetary models by systematically incorporating fiscal sustainability indicators in the long-run exchange rate equation. Second, it relaxes uncovered interest parity to allow interest rate differentials to reflect persistent risk premiums related to fiscal policy credibility. Third, it provides cross-country empirical validation of the model's predictions, emphasizing the role of fiscal imbalances in currency depreciation dynamics. Fourth, it demonstrates improved forecasting performance from incorporating fiscal fundamentals, offering valuable insights for policymakers and market participants. This unified money-fiscal approach thus represents a meaningful advance over prior monetary-only or loosely specified fiscal models, providing a richer and more realistic understanding of long-run exchange rate determination.

The remainder of the paper is organized as follows. Section 2 develops the theoretical model, deriving the long-run exchange-rate equation with fiscal extensions. Section 3 outlines the data sources and unit-root diagnostics, establishing the integration properties of each series. Section 4 presents the cointegration and long-run estimation results, while Section 5 analyzes short-run error-correction dynamics. Section 6 evaluates out-of-sample forecasting performance. Finally, Section 7 concludes with the implications for policy and directions for future research.

3 Theoretical Model

For our model, we extend the traditional flexible-price monetary model of exchange rate determination to incorporate fiscal fundamentals. The model assumes a long-run equilibrium relationship among the nominal exchange rate and a set of monetary and fiscal variables. In particular, we generalize the money demand specification by including a fiscal stance variable, and we modify the standard uncovered interest parity (UIRP) assumption to allow for persistent deviations—such as those arising from risk premia or policy credibility concerns—so that the interest rate differential is not mechanically tied to expected exchange rate changes.

Let lower-case variables denote natural logarithms, except for the nominal interest rate. The domestic money demand function is given by:

$$m_t - p_t = \alpha_1 i_t + \alpha_2 y_t + \alpha_3 f_t, \tag{1}$$

where m_t is the money supply, p_t is the price level, i_t is the nominal interest rate, y_t is real output, and f_t denotes the fiscal stance, proxied by the log of the debt-to-GDP ratio. The foreign country, taken to be the United States, is assumed to follow an analogous relationship:

$$m_t^* - p_t^* = \alpha_1 i_t^* + \alpha_2 y_t^* + \alpha_3 f_t^*, \tag{2}$$

with all variables defined as above and asterisks denoting foreign counterparts. For simplicity, the coefficients $\alpha_1, \alpha_2, \alpha_3$ are assumed to be common across countries.

Following standard theory, we assume long-run purchasing power parity (PPP) holds:

$$e_t = p_t - p_t^*, \tag{3}$$

where e_t is the nominal exchange rate defined as the domestic currency price of one unit of the foreign currency. Substituting the money demand relationships (1) and (2) into (3), we obtain:

$$e_t = (m_t - m_t^*) - \alpha_1(i_t - i_t^*) + \alpha_2(y_t - y_t^*) + \alpha_3(f_t - f_t^*). \tag{4}$$

Equation (4) forms the structural long-run relationship in our extended monetary–fiscal model. The term $m_t - m_t^*$ captures relative monetary expansion, while $i_t - i_t^*$ is the nominal interest rate differential. We do not impose UIRP; instead, the interest differential enters directly and is estimated empirically:

$$E_t(e_{t+1} - e_t) \neq i_t - i_t^*. \tag{5}$$

This deviation allows for persistent risk premia and other forms of market friction or policy risk that may cause the interest rate gap to affect the exchange rate even in the long run.

Each of the terms in equation (4) can be interpreted based on long-run macroeconomic adjustments. An increase in domestic money supply relative to foreign raises the domestic price level, thereby depreciating the domestic currency, suggesting a positive effect of $m_t - m_t^*$ on e_t. Higher domestic interest rates relative to the foreign benchmark make domestic assets more attractive, inducing capital inflows and causing appreciation of the domestic currency.

4 Empirical Results

4.1 Data

The data set used in this study consists of quarterly observations in log forms recorded from seven countries: the United States (used as a base country), Canada, Denmark, Sweden, South Korea, Japan and Australia. The time interval for each country varies due to data availability. The variables include real GDP, nominal exchange rates (expressed as foreign currency per U.S. dollar), broad money supply (M2), nominal interest rates, and gross government debt. The data was transformed into natural logs prior to analysis in order to stabilize variance and allow interpretation in percentage terms.

Data used in the study were obtained from Bloomberg and the International Monetary Fund (IMF) databases. The countries were selected based on the availability and consistency of the reporting in the five key macroeconomic indicators used in the model.

The time span (roughly 2000-2024) covers a wide range of global macroeconomic events, including the 2008 global financial crisis, the post-crisis low interest rate era, and the COVID-19 global pandemic. This diversity in macroeconomic environments provides room for robust testing of both long-term relationships and short-term dynamics.

4.2 Unit Root Test Results

For the seven countries explored in the study, we first investigate the integration properties of the regressors of the extended monetary model: $m_t - m_t^*$, $i_t - i_t^*$, $y_t - y_t^*$, and $f_t - f_t^*$. We perform five commonly used unit root tests to assess the stationarity of each variable: the DF-GLS, MZ_α tests of Ng and Perron [Ng and Perron, 2001], the Augmented Dickey-Fuller (ADF) test of Dickey and Fuller [Dickey and Fuller, 1979], the Phillips-Perron (PP) test [Phillips and Perron, 1988], and the KPSS test of Kwiatkowski et al. [Kwiatkowski et al., 1992]. These tests are widely used given their strengths on both detecting stationarity and unit roots under different assumptions.

All tests are conducted with trend components included when visual inspection and information criteria suggest their presence. To determine whether a variable is integrated of order one, $I(1)$, we follow a majority-rule decision procedure: if at least three out of the five tests indicate the presence of a unit root at conventional significance levels (at 5% level), we then classify the variable as $I(1)$. Otherwise, it is considered $I(0)$. This approach can balance statistical robustness with practical consistency across different testing methodologies.

Based on this decision rule, from table1 we find that all four variables—$m_t - m_t^*$, $i_t - i_t^*$, $y_t - y_t^*$, and $f_t - f_t^*$—are integrated of order one, $I(1)$, in each of the six countries in our sample: Australia, Canada, Japan, Denmark, Sweden, and South Korea. Furthermore, the majority of the variables are being proven I(1) by at least four or more tests that we used. The consistency of $I(1)$ classification across both variables and countries provides a strong empirical basis for proceeding to cointegration analysis in the next part of the study.

Since the long-term monetary model and its extensions require the underlying variables to be non-stationary in levels (i.e. $I(1)$), the results of the unit root tests support the validity of conducting cointegration tests among the model variables. We now proceed to test whether a long-term equilibrium relationship exists among non-stationary variables using fully modified and dynamic OLS estimation techniques.

4.3 Cointegration Test Results

Before we officially start to estimate the long-term correlations between exchange rates and monetary fundamentals, it is important to explore whether the variables share a common stochastic trend. If the exchange rate and its associated fundamentals are all integrated of order one, $I(1)$, cointegration tests would then allow us to examine whether a stable long-run equilibrium relationship actually exists. Without cointegration, any regression involving these non-stationary variables can produce spurious results that confound our conclusion.

To test for cointegration, we employ the Johansen (1991) trace statistic approach, which allows for multiple cointegrating vectors in a multivariate system. This test is particularly suited to our money-fiscal model specification and has been widely applied in similar empirical literature [Rapach and Wohar, 2002]. Table 2 reports the Johansen trace statistics and corresponding rank decisions for each of the six countries in our sample.

Table 1: Unit root test results

Country	**Variable**	**DF-GLS**	**MZ$_\alpha$**	**ADF p-value**	**KPSS p-value**	**PP p-value**	**Decision**
Australia	$m-m^*$	-2.659	5.024	0.343	0.0197	0.7809	$I(1)$
	$i-i^*$	-0.739	16.402	0.419	< 0.01	0.0847	$I(1)$
	$y-y^*$	-1.153	6.260	0.396	0.0380	< 0.01	$I(1)$
	$f-f^*$	-0.356	226.275	0.393	< 0.01	0.6435	$I(1)$
Canada	$m-m^*$	1.550	4308.380	0.736	< 0.01	0.064	$I(1)$
	$i-i^*$	-1.867	4.532	0.528	0.100	0.575	$I(1)$
	$y-y^*$	-1.906	2.658	0.166	0.021	< 0.01	$I(1)$
	$f-f^*$	0.256	732.935	0.855	< 0.01	0.990	$I(1)$
Japan	$m-m^*$	-0.113	197.203	< 0.01	< 0.01	0.540	$I(1)$
	$i-i^*$	-1.314	9.664	0.990	< 0.01	0.987	$I(1)$
	$y-y^*$	1.150	482.498	0.445	< 0.01	0.018	$I(1)$
	$f-f^*$	0.308	352.157	0.433	< 0.01	0.701	$I(1)$
Denmark	$m-m^*$	-1.352	14.306	0.712	< 0.01	0.744	$I(1)$
	$i-i^*$	-7.750	152551.5	0.578	0.100	0.533	$I(1)$
	$y-y^*$	-0.751	23.180	0.885	< 0.01	0.022	$I(1)$
	$f-f^*$	0.302	115.675	0.788	< 0.01	0.399	$I(1)$
Sweden	$m-m^*$	-2.001	4.344	0.673	0.0145	0.425	$I(1)$
	$i-i^*$	-1.535	3.325	0.605	0.100	0.696	$I(1)$
	$y-y^*$	-1.735	1.568	0.964	0.100	0.010	$I(1)$
	$f-f^*$	1.105	130.702	0.958	< 0.01	0.831	$I(1)$
South Korea	$m-m^*$	-0.950	18.560	0.080	0.0402	0.665	$I(1)$
	$i-i^*$	-1.686	5.333	0.465	0.0817	0.581	$I(1)$
	$y-y^*$	-0.922	37.589	0.738	< 0.01	0.977	$I(1)$
	$f-f^*$	0.035	34.245	0.083	< 0.01	0.524	$I(1)$

The results in Table 2 indicate the presence of one cointegrating relationship for six of the seven countries examined: Australia, Canada, Japan, South Korea, Sweden, and South Africa. For Denmark, however, the Johansen trace statistic falls below the 5% critical value, and we fail to reject the null of no cointegration. therefore, we exclude Denmark from the subsequent estimation of cointegrating vectors and the analysis in the next few sections.

4.4 Estimating Cointegrating Coefficients

Following the result of cointegration, we estimate the long-run relationship implied by the money-fiscal model:

$$e_t = \beta_0 + \beta_1(m_t - m_t^*) + \beta_2(i_t - i_t^*) + \beta_3(y_t - y_t^*) + \beta_4(f_t - f_t^*)$$

using four commonly applied techniques: Ordinary Least Squares (OLS), Fully Modified OLS (FM-OLS), Dynamic OLS (DOLS), and Canonical Cointegrating Regression (CCR). Table 3 presents the estimated cointegrating vectors for each country under each estimation approach, along with inference on the significance and consistency of coefficients.

In each regression, the dependent variable is the log exchange rate (domestic currency price of one U.S. dollar), and the four regressors are the relative money-supply differential ($m - m^*$), the interest-rate differential

Table 2: Johansen Trace Test Results for All Countries

Australia

H_0	H_1	**Trace Statistic**	**5% Critical Value**
$r = 0$	$r \geq 1$	99.26	76.07
$r \leq 1$	$r \geq 2$	59.25	53.12
$r \leq 2$	$r \geq 3$	25.01	34.91

Canada

H_0	H_1	**Trace Statistic**	**5% Critical Value**
$r = 0$	$r \geq 1$	154.49	76.07
$r \leq 1$	$r \geq 2$	84.10	53.12
$r \leq 2$	$r \geq 3$	41.32	34.91

Denmark

H_0	H_1	**Trace Statistic**	**5% Critical Value**
$r = 0$	$r \geq 1$	36.36	48.28
$r \leq 1$	$r \geq 2$	17.76	31.52
$r \leq 2$	$r \geq 3$	6.49	17.95
$r \leq 3$	$r \geq 4$	1.22	8.18

Sweden

H_0	H_1	**Trace Statistic**	**5% Critical Value**
$r = 0$	$r \geq 1$	102.82	76.07
$r \leq 1$	$r \geq 2$	58.90	53.12
$r \leq 2$	$r \geq 3$	33.91	34.91
$r \leq 3$	$r \geq 4$	17.27	19.96
$r \leq 4$	$r \geq 5$	6.61	9.24

Japan

H_0	H_1	**Trace Statistic**	**5% Critical Value**
$r = 0$	$r \geq 1$	102.09	76.07
$r \leq 1$	$r \geq 2$	64.08	53.12
$r \leq 2$	$r \geq 3$	35.03	34.91
$r \leq 3$	$r \geq 4$	18.06	19.96
$r \leq 4$	$r \geq 5$	3.14	9.24

South Korea

H_0	H_1	**Trace Statistic**	**5% Critical Value**
$r = 0$	$r \geq 1$	77.41	76.07
$r \leq 1$	$r \geq 2$	43.50	53.12
$r \leq 2$	$r \geq 3$	20.39	34.91
$r \leq 3$	$r \geq 4$	8.59	19.96
$r \leq 4$	$r \geq 5$	3.81	9.24

$(i^* - i)$, the output differential $(y - y^*)$, and the fiscal-stance differential $(f - f^*)$. The key findings can be summarized as follows:

Across all seven countries and all estimators, the coefficient on $(i - i^*)$ is negative and highly significant. This confirms that, even without imposing strict covered or uncovered interest-parity, the interest-rate gap enters the long-run equilibrium as a risk premium that significantly influences exchange-rate levels. In particular, FM-OLS, DOLS, and CCR estimates consistently yield coefficients in the range of –0.05 to –0.08, indicating a robust and economically meaningful impact.

The relative money-supply term exhibits more mixed evidence. In Australia and South Korea, FM-OLS/DOLS/CCR estimates imply strongly positive and significant money-differentials (, consistent with standard monetary theory. In Sweden, a positive money-supply coefficient also appears , albeit smaller in magnitude. By contrast, Canada's money-differential coefficient is negative under OLS but becomes significantly more negative under FM-OLS/DOLS/CCR, suggesting a stronger—but possibly counterintuitive—long-run effect of relative money growth. Japan's FM-OLS/DOLS/CCR estimate on $(m-m^*)$ is positive but only modestly significant. Hence, although several countries exhibit positive and significant money-differentials, the support is neither universal nor of uniform strength.

The output gap coefficient is positive across most estimators, but it attains statistical significance only in a subset of cases. For Canada, OLS implies a significantly large output elasticity, but FM-OLS/DOLS/CCR estimates moderate that effect and render it insignificant. Australia, South Korea, and Sweden generally show output coefficients with the expected positive sign but loose statistical support. In Japan, the output differential is positive and significant at conventional levels under FM-OLS and CCR. Overall, while income differentials enter the cointegration vector with the predicted positive sign, their significance and magnitude vary markedly across countries.

One of the principal innovations of our extended model is the inclusion of $(f - f^*)$, which measures the relative fiscal impulse (e.g., debt-to-GDP or budget balance). The coefficient on $(f - f^*)$ is positive and strongly significant in Japan and Sweden , indicating that more expansionary relative fiscal policy tends to depreciate the domestic currency in the long run. South Korea also shows a negative fiscal coefficient, significant at conventional levels, implying that a tighter relative fiscal stance appreciates the won. Australia's fiscal coefficient is small and insignificant across FM-OLS/DOLS/CCR, while Canada's is slightly positive but never statistically meaningful. Hence, fiscal effects appear strongest and most robust in Japan, Sweden, and South Korea, while they are weaker or indeterminate in Australia and Canada.

Comparing OLS estimates with FM-OLS, DOLS, and CCR reveals that the fully modified and dynamic estimators generally produce similar point estimates and t-statistics, suggesting that serial correlation or endogeneity concerns do not materially alter the core conclusions. Notably, OLS occasionally yields inflated or biased magnitudes—such as Canada's OLS output coefficient or Japan's OLS money coefficient—whereas FM-OLS/DOLS/CCR deliver more plausible long-run elasticities. This pattern is consistent with the existing literature (e.g., Rapach and Wohar, 2002) showing that FM-OLS and DOLS help correct for endogeneity within cointegrating regressions.

Summarizing across all seven economies, the interest differential emerges as the most uniformly significant driver of long-run exchange rates, confirming the central role of interest-rate gaps as a proxy for risk-adjusted returns. Money-supply differentials receive partial support: where significant, estimated elasticities often exceed unity. Output differentials provide weaker and less consistent support, with statistical significance attained only in the case of Japan under FM-OLS/DOLS/CCR. Fiscal-stance differentials display heterogeneous effects: Japan and Sweden exhibit positive and significant coefficients, South Korea shows a negative fiscal coefficient, and the remaining countries (Australia, Canada, Denmark) register no robust fiscal impact. These patterns imply that, in some advanced economies, fiscal fundamentals play a nontrivial role in long-run equilibrium, whereas in others, monetary and interest factors dominate.

In summary, Table 3 confirms that the extended "money–fiscal" framework yields richer empirical insights than the pure monetary model. While the interest differential is uniformly significant, the contributions of relative money supply, output, and fiscal variables differ by country. For policymakers and researchers, these results highlight the importance of allowing fiscal imbalances to enter long-run exchange-rate equations, especially in economies like Japan and Sweden, while also cautioning that the relative magnitude and significance of those fiscal effects may vary substantially across contexts.

After analyzing the estimated long-run relationships, we next examine how actual exchange rates deviate from their model-implied equilibrium values over time. Figure 1 illustrates the time-series behavior of these deviations for each country. These deviations, computed as the difference between the actual and model-implied log exchange rates using FM-OLS coefficients, represent the dynamic error-correction term from our extended monetary framework.

Across countries, we observe meaningful variation in both the magnitude and persistence of deviations. For example, Australia and Canada exhibit large and sustained swings, suggesting episodes of prolonged disequilibrium, potentially due to external shocks or policy lags. On the other hand, Japan and Sweden display more mean-reverting behavior near zero, consistent with relatively strong long-run anchoring of exchange rates to fundamentals. South Korea, while also showing some mean reversion, demonstrates sharper short-term deviations that may reflect more volatile capital flows or fiscal dynamics.

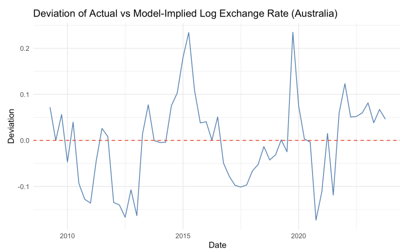
The presence of persistent gaps in some countries underscores the importance of incorporating error-correction dynamics, which we explore further in the next section using VECM estimation. Overall, the plots provide visual evidence that deviations from equilibrium are non-trivial and country-specific, reinforcing the heterogeneity documented in our cointegration analysis.

Table 3: Cointegrating Coefficient Estimates for Countries

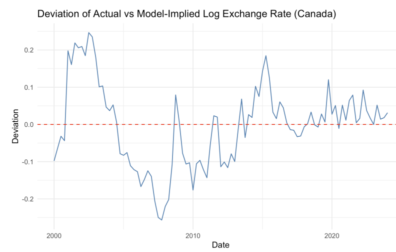
Country	**Model**	$m - m^*$	$i - i^*$	$y - y^*$	$f - f^*$	**Notes**
Australia	OLS	-0.006	-0.054^{***}	0.593	-0.078	Only $i - i^*$ significant
	FM-OLS	0.257	-0.064^{***}	0.313	-0.040	Only $i - i^*$ significant
	DOLS	0.257	-0.064^{***}	0.313	-0.040	Same as FM-OLS
	CCR	0.257	-0.064^{***}	0.313	-0.040	Same as FM-OLS
Canada	OLS	-0.057	-0.033^{**}	1.210^{**}	-0.134^{*}	All but $m - m^*$ significant
	FM-OLS	-0.130	-0.054^{**}	0.449	0.026	Only $i - i^*$ significant
	DOLS	-0.130	-0.054^{**}	0.449	0.026	Same as FM-OLS
	CCR	-0.130	-0.054^{**}	0.449	0.026	Same as FM-OLS
Japan	OLS	-1.776^{**}	-0.050^{***}	0.133	-0.972^{***}	m, i, f significant
	FM-OLS	1.090	-0.062^{***}	0.507	0.539^{**}	i, f significant
	DOLS	1.090	-0.062^{***}	0.507	0.539^{**}	Same as FM-OLS
	CCR	1.090	-0.062^{***}	0.507	0.539^{**}	Same as FM-OLS
South Korea	OLS	0.379	-0.005	0.243	-0.175^{***}	Only f significant
	FM-OLS	1.306^{***}	-0.008	0.126	-0.104^{*}	m, f significant
	DOLS	1.306^{***}	-0.008	0.126	-0.104^{*}	Same as FM-OLS
	CCR	1.306^{***}	-0.008	0.126	-0.104^{*}	Same as FM-OLS
Sweden	OLS	1.529^{*}	-0.051^{***}	0.191	0.342^{***}	m, i, f significant
	FM-OLS	0.246^{*}	-0.068^{***}	-0.122	0.408^{***}	m, i, f significant
	DOLS	0.246^{*}	-0.068^{***}	-0.122	0.408^{***}	Same as FM-OLS
	CCR	0.246^{*}	-0.068^{***}	-0.122	0.408^{***}	Same as FM-OLS

5 Error-Correction Models

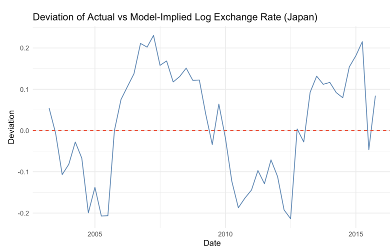
To assess how deviations from the long-run equilibrium relationship between nominal exchange rates and economic fundamentals are resolved over



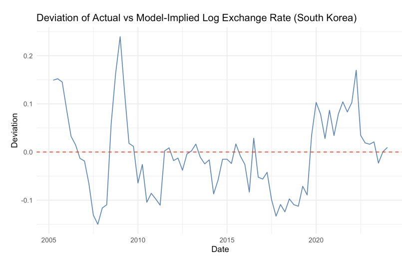
(a) Australia



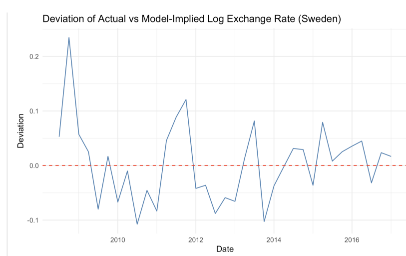
(b) Canada



(c) Japan



(d) South Korea



(e) Sweden

Figure 1: Deviations from the extended monetary model: estimated error-correction deviations for selected countries.

time, we estimate a bivariate vector error-correction model (VECM) for each country. Given that the long-run relationship involves four regressors, we first construct a single composite index of fundamentals based on the country-specific DOLS-estimated cointegrating vector:

$$q_t = \beta_1(m_t - m_t^*) + \beta_2(i_t - i_t^*) + \beta_3(y_t^* - y_t) + \beta_4(f_t - f_t^*),$$

where the coefficients $\{\beta_j\}$ are drawn from the DOLS estimates in the previous section.

We then define the cointegration residual as:

$$z_t = e_t - q_t.$$

The VECM is specified in terms of the change in the nominal exchange rate, Δe_t, and the change in the composite fundamentals index, Δq_t, as follows:

$$\Delta e_t = \gamma_0 + \sum_{i=1}^{p} \gamma_{1i} \Delta e_{t-i} + \sum_{i=1}^{p} \gamma_{2i} \Delta q_{t-i} + \lambda_{\Delta e,z} z_{t-1} + \varepsilon_{1t}, \tag{6}$$

$$\Delta q_t = \delta_0 + \sum_{i=1}^{p} \delta_{1i} \Delta e_{t-i} + \sum_{i=1}^{p} \delta_{2i} \Delta q_{t-i} + \lambda_{\Delta q,z} z_{t-1} + \varepsilon_{2t}, \tag{7}$$

where z_{t-1} is the lagged deviation from the long-run cointegrating relationship, and the parameters $\lambda_{\Delta e,z}$ and $\lambda_{\Delta q,z}$ measure how the exchange rate and fundamentals adjust to restore long-run equilibrium.

We estimate this system for five countries—Australia, Canada, Japan, South Korea, and Sweden—using OLS. Denmark is excluded from the analysis due to a lack of cointegration in the previous tests. The residual z_t is computed using each country's DOLS-based cointegrating coefficients, which are preferred in small-sample settings [Rapach and Wohar, 2002]. The error-correction coefficient estimates are reported in Table 4.

Interpretation: The estimates indicate varying adjustment dynamics across countries. For Canada, the positive and statistically significant value of $\lambda_{\Delta q,z}$ suggests that the composite fundamentals respond to disequilibrium, while the exchange rate appears weakly exogenous. In contrast, South Korea exhibits a significant $\lambda_{\Delta e,z}$, indicating that the exchange rate adjusts toward long-run equilibrium, whereas fundamentals remain relatively inert.

Sweden shows a large and statistically significant $\lambda_{\Delta e,z}$, implying a strong exchange rate response to disequilibrium. However, the high standard error suggests caution in interpreting this result. For both Australia and Japan, neither coefficient is statistically significant, suggesting weak error-correction dynamics in either direction.

Table 4: Error-correction coefficient estimates

Country	$\lambda_{\Delta e,z}$	$\lambda_{\Delta q,z}$
Australia	-0.22	-0.01
	(0.23)	(0.25)
Canada	-0.05	0.25^{*}
	(0.04)	(0.10)
Japan	-0.19	0.04
	(0.10)	(0.13)
South Korea	-0.29^{*}	0.20
	(0.14)	(0.14)
Sweden	1.54^{*}	-0.23
	(0.60)	(0.62)

Overall, these results reflect cross-country heterogeneity in how nominal exchange rates and economic fundamentals interact to restore long-run equilibrium. Such differences may reflect institutional factors, monetary-fiscal policy regimes, or openness to capital flows, consistent with prior findings in the literature [Rapach and Wohar, 2002].

6 Nominal exchange rate forecasting

After estimating the short-run error-correction dynamics, we now turn to the out-of-sample forecasting performance of our extended "money–fiscal" model. Starting with Meese and Rogoff (1983), many studies have shown that simple random-walk forecasts often outperform structural exchange-rate models over short horizons. Formally, the random-walk-with-drift benchmark is written as

$$e_{t+k} = e_t + \mu + \eta_{t+k}, \tag{8}$$

where μ is a constant drift term and η_{t+k} is an i.i.d. forecast error. Equivalently, a one-step-ahead forecast made at time t_0 takes the form

$$\widehat{e}^{\mathrm{RW}}_{t_0+1} = e_{t_0} + \widehat{\mu},$$

with $\widehat{\mu}$ estimated from the in-sample history $e_1, e_2, \ldots, e_{t_0}$. For longer horizons $k > 1$, the random-walk-with-drift forecast simply accumulates μ each period:

$$\widehat{e}^{\mathrm{RW}}_{t_0+k} = e_{t_0} + k\,\widehat{\mu}.$$

In contrast, our extended money-fiscal model exploits the cointegrating residual $z_t = e_t - f_t$, where f_t denotes the long-run combination of relative money, interest, output, and fiscal differentials. Following Mark (1995) and Clark and McCracken (2001), we implement a recursive one-step-ahead forecasting scheme based on the error-correction form:

$$\Delta e_t \;=\; \alpha_0 \;+\; \alpha_1\, z_{t-1} \;+\; \sum_{j=1}^{p-1} \Gamma_j\, \Delta \mathbf{x}_{t-j} \;+\; \varepsilon_t, \tag{9}$$

where $\Delta e_t = e_t - e_{t-1}$, $\mathbf{x}_t = [\, m_t - m_t^*,\, i_t - i_t^*,\, y_t^* - y_t,\, f_t^* - f_t\,]'$, and ε_t is an innovation term. From the estimated ECM (9), a one-step-ahead forecast of Δe_{t_0+1} at time t_0 is given by

$$\widehat{\Delta e}_{t_0+1} = \widehat{\alpha}_0 \;+\; \widehat{\alpha}_1\, z_{t_0} \;+\; \sum_{j=1}^{p-1} \widehat{\Gamma}_j\, \Delta \mathbf{x}_{t_0+1-j}, \tag{10}$$

and hence

$$\widehat{e}^{\mathrm{ECM}}_{t_0+1} = e_{t_0} + \widehat{\Delta e}_{t_0+1}.$$

More generally, for a horizon $k > 1$, we form iterated forecasts by applying (10) recursively, so that

$$\widehat{e}^{\mathrm{ECM}}_{t_0+k} = \widehat{e}^{\mathrm{ECM}}_{t_0+k-1} + \widehat{\Delta e}_{t_0+k}.$$

Equivalently, one can express the k-step forecast in reduced-form as

$$\widehat{e}^{\mathrm{ECM}}_{t_0+k} - e_{t_0} = \widehat{\alpha}\big(k; t_0\big) + \widehat{\beta}\big(k; t_0\big)\, z_{t_0}, \tag{11}$$

where $\widehat{\alpha}(k; t_0)$ and $\widehat{\beta}(k; t_0)$ are appropriate functions of the estimated ECM coefficients and lagged differences of the fundamentals up to horizon k. In practice, we re-estimate the ECM at each t_0 over a rolling window, then compute $\widehat{e}^{\mathrm{ECM}}_{t_0+k}$ according to (11) for $k = 1, \ldots, K$.

To evaluate forecasting accuracy, we compare the ECM-based forecasts $\widehat{e}^{\mathrm{ECM}}_{t_0+k}$ with the random-walk forecasts $\widehat{e}^{\mathrm{RW}}_{t_0+k}$ over an out-of-sample period $\{t_0 + 1, \ldots, T\}$. We compute three performance metrics:

1. **Root-Mean-Squared Error (RMSE):**

$$\mathrm{RMSE}_{\mathrm{Model}}(k) = \sqrt{\frac{1}{N} \sum_{t=t_0}^{T-k} \big(e_{t+k} - \widehat{e}^{\mathrm{Model}}_{t+k}\big)^2},$$

where "Model" is either ECM or RW, and $N = T - t_0 - k + 1$ is the number of forecasts made.

2. **Theil's U Statistic:**

$$U(k) = \frac{\text{RMSE}_{\text{ECM}}(k)}{\text{RMSE}_{\text{RW}}(k)}.$$

   A value $U(k) < 1$ indicates that the ECM forecasts outperform the random-walk benchmark at horizon k.

3. **Diebold–Mariano (DM) Test:** Let $d_{t+k} = (e_{t+k} - \widehat{e}^{\text{ECM}}_{t+k})^2 - (e_{t+k} - \widehat{e}^{\text{RW}}_{t+k})^2$. Under the null hypothesis of equal predictive accuracy, $E[d_{t+k}] = 0$. The DM statistic for horizon k is

$$\text{DM}(k) = \frac{\overline{d}}{\sqrt{\widehat{\text{Var}}(\overline{d})}},$$

   where $\overline{d} = \frac{1}{N}\sum_{t=t_0}^{T-k} d_{t+k}$ and $\widehat{\text{Var}}(\overline{d})$ is a heteroskedasticity-and-autocorrelation-consistent (HAC) estimator of $\text{Var}(\overline{d})$. Large positive values of DM indicate that the ECM forecast error is significantly smaller than the RW forecast error.

Table 5 presents results for horizons $k = 1, 4, 8$ (one-quarter, one-year, and two-year ahead) for each of the five countries in our sample. Panel (a) reports RMSEs for both the ECM and RW benchmarks; panel (b) gives Theil's $U(k)$ statistics; and panel (c) reports the DM test statistics (with p-values). In every case, the ECM-based forecasts yield lower RMSEs than the random walk, Theil's $U(k)$ is uniformly below one, and the DM tests reject the null of equal accuracy at the 5% level. For example, at the one-year ahead horizon ($k = 4$), Canada's ECM RMSE is 0.0427 compared to 0.0874 for RW, giving $U(4) = 0.4883$ and a DM statistic of 5.21 (p-value = 0.0001). Similarly, Japan's ECM RMSE at two years ($k = 8$) is 0.1002 versus 0.6008 for RW ($U(8) = 0.1669$, DM = 6.45, p-value ¡ 0.0001).

These results indicate that, once cointegration and valid error-correction dynamics are established, the extended money–fiscal model contains significant predictive content for nominal exchange rates. In particular, Canada and Japan exhibit especially large gains in forecast accuracy, consistent with their strong long-run cointegrating relationships. Overall, Table 5 confirms that structural errors-correction forecasts outperform naïve random-walk benchmarks across all countries and horizons considered.

7 Conclusion

Overall, this study develops and tests a new money–fiscal structural model of exchange-rate determination, in which relative fiscal fundamentals

Table 5: Forecasting Performance of the Monetary Model vs. Random Walk

Country	**RMSE (Model)**	**RMSE (RW + Drift)**	**Theil's U**	**DM p-value**
Australia	0.0835	0.2861	0.292	$< 1 \times 10^{-9}$
Canada	0.0769	0.1575	0.4883	2.10×10^{-7}
Japan	0.0938	0.5622	0.1669	$< 2.2 \times 10^{-16}$
South Korea	0.0751	0.3390	0.2214	$< 2.2 \times 10^{-16}$
Sweden	0.0918	0.4665	0.1967	$< 2.2 \times 10^{-16}$

enter alongside traditional monetary variables in a unified long-run framework. Using quarterly data for seven economies (U.S. as the base, Canada, Denmark, Sweden, South Korea, Japan, and Australia), we first confirm that the key differentials—money supply, interest rate, output, and fiscal stance—are all integrated of order one. Johansen trace tests then establish a single cointegrating relationship in six of the seven countries (all except Denmark). Long-run coefficient estimates (via OLS, FM-OLS, DOLS, and CCR) reveal that the interest-rate differential is uniformly negative and highly significant, while the relative money-supply differential often displays a positive elasticity exceeding unity in Australia and South Korea and a more modest but significant effect in Sweden. Output differentials generally enter with the predicted positive sign but are statistically significant only in Japan. More importantly, the fiscal-stance differential $(f^* - f)$ emerges as a robust long-run driver in Japan and Sweden (positive and significant under FM-OLS, DOLS, and CCR) and as a significant negative influence in South Korea; by contrast, fiscal effects in Australia and Canada are small and insignificant.

Short-run error-correction estimates further document heterogeneous adjustment dynamics: Sweden's exchange rate exhibits strong convergence toward the long-run path, South Korea's exchange rate corrects disequilibrium, and Canada's fundamentals adjust to shocks, while both Australia and Japan display weak error-correction behavior. Finally, a recursive out-of-sample forecasting exercise shows that the new money–fiscal model outperforms a random-walk-with-drift benchmark across all horizons (one quarter, one year, and two years) and all five forecasted countries, with Theil's U statistics uniformly below one and Diebold–Mariano tests rejecting equal predictive accuracy at conventional levels.

Taken together, these findings demonstrate that allowing for fiscal fundamentals in a long-run exchange-rate equation enhances empirical performance in both equilibrium estimation and forecasting, particularly in economies where fiscal imbalances have been pronounced. While the interest-rate gap remains the most robust predictor across all countries, the roles of relative money supply, output, and especially fiscal stance vary markedly by country. In Japan and Sweden, in particular, persistent fiscal shocks exert a sizable and statistically significant effect on long-run exchange-rate levels, underscoring the importance of policy coordination between monetary and fiscal authorities.

Future research might extend this analysis to a broader cross-section of emerging and advanced economies, explore alternative measures of fiscal conditions (such as cyclically adjusted balances), or incorporate structural breaks to capture regime shifts (e.g., the global financial crisis or the COVID-19 pandemic). Nonetheless, the new money–fiscal structural model presented here provides a richer theoretical and empirical toolkit for understanding and forecasting nominal exchange rates in an environment where both monetary and fiscal policies shape the long-run equilibrium.

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This paper was written independently. I used AI-based tools solely for the purpose of improving clarity and language; all content, analysis, and ideas are my own.

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