Understanding Inflation: Concepts, Changes, and Mathematical Insights

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Abstract

This article explains the concept of inflation, clarifies common misconceptions, and explores how to measure changes in inflation such as increasing and accelerating inflation. We provide mathematical derivations using derivatives, real-world examples, and practical rules for analysis. We also include a formal theorem on the relationship between accelerating and increasing inflation.

1 Introduction

Inflation is one of those seemingly simple concepts with major impacts on our lives. Many people say, "Prices have gone up, inflation is everywhere. I can't buy this or that; I can't afford to pay for basic needs. I feel poorer and more miserable."

Yet, understanding the dynamics of this single variable can be confusing. This article explains what inflation is—and what it is not—and how to interpret its changes over time.

Although inflation is usually reported at discrete intervals (monthly or quarterly), for analytical clarity we will assume it is observed continuously. This assumption introduces no loss of generality but helps us derive rigorous insights.

2 Discrete vs. Continuous Time

If the general price level were reported at a very high frequency (every millisecond, second, minute, or hour), as in the case of stock prices, we would say it is recorded in continuous time. Mathematically, time $t \in \mathbb{R}$.

In reality, price levels are reported at regular intervals—monthly or quarterly—so we usually deal with discrete time. This is why inflation is reported as month-on-month, quarter-on-quarter, or year-on-year growth in prices.

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3 What Is Inflation?

Inflation is a fundamental economic concept describing the rate of change (in percentage terms) of the overall price level in an economy, again expressed as a percentage, not just absolute change in prices over time.

Important Clarification

Inflation is **not** simply the first derivative of the price level with respect to time.

Why? Because inflation measures the *relative* or *percentage* change in prices, not just the absolute change. Formally, in continuous time (well, real life price level of a country is reported in discrete time, e.g. monthly, quarterly, etc), inflation at time t is:

$$\pi(t) = \frac{1}{P(t)}\frac{dP(t)}{dt}$$

Where:

- $P(t)$: price level at time t,
- $\frac{dP(t)}{dt}$: rate of change of the price level,
- $\frac{1}{P(t)}$: converts absolute change into percentage change.

This makes inflation a normalized derivative. Note that we have adopted a continuous time framework for ease of exposition, but the argument remains true in discrete time.

Key Insight

Even if the absolute rate of change $\frac{dP}{dt}$ increases, inflation can still fall if $P(t)$ increases even faster. Therefore, inflation is not simply the derivative of price level.

4 How Inflation Changes: Increasing vs. Accelerating

Increasing inflation means the inflation rate is rising over time (e.g., from 2% to 4%). **Accelerating inflation** means the rate at which inflation is increasing is itself rising (e.g., from 2% to 3%, then to 5%).

Mathematical Formulation

Let:

$$\pi(t) = \frac{1}{P(t)}\frac{dP(t)}{dt}$$

Increasing Inflation

Inflation is increasing if:

$$\frac{d\pi}{dt} > 0$$

From chain rule:

$$\frac{d\pi}{dt} = \frac{d}{dt}\left(\frac{1}{P(t)}\frac{dP(t)}{dt}\right) = -\frac{1}{P(t)^2}\left(\frac{dP(t)}{dt}\right)^2 + \frac{1}{P(t)} \cdot \frac{d^2P(t)}{dt^2}$$

Hence, increasing inflation requires:

$$\frac{d^2P}{dt^2} > \pi(t) \cdot \frac{dP}{dt}$$

Accelerating Inflation

Accelerating inflation means:

$$\frac{d^2\pi}{dt^2} > 0$$

This depends on third derivatives of price and is more complex.

5 Real-World Example: Inflation Over Six Years

Year	Inflation π_t (%)	Change ($\pi_t - \pi_{t-1}$)	Increasing?	Rate Change (%)	Accelerating?
2018	2.0	—	—	—	—
2019	2.5	+0.5	Yes	25	—
2020	3.0	+0.5	Yes	20	No
2021	4.0	+1.0	Yes	33	Yes
2022	5.5	+1.5	Yes	37.5	Yes
2023	5.0	–0.5	No	–9	No

Table 1: Example of increasing and accelerating inflation, 2018–2023

Interpretation

- Inflation increased each year from 2018 to 2022.
- The rate of increase was modest in 2020 but accelerated in 2021 and 2022.
- In 2023, inflation declined—neither increasing nor accelerating.

6 Rules of Thumb

Let $\pi_{t-1}, \pi_t, \pi_{t+1}$ denote inflation rates in three consecutive years.

1. Increasing Inflation (Level Test)

$$\pi_t - \pi_{t-1} > 0 \quad \text{and} \quad \pi_{t+1} - \pi_t > 0$$

2. Relative Growth in Inflation

$$\frac{\pi_{t+1}}{\pi_t} - 1 \quad \text{and} \quad \frac{\pi_t}{\pi_{t-1}} - 1$$

3. Accelerating Inflation

$$\left(\frac{\pi_{t+1}}{\pi_t} - 1\right) \gg \left(\frac{\pi_t}{\pi_{t-1}} - 1\right)$$

7 Theorem: Accelerating vs. Increasing Inflation

Theorem. If inflation is accelerating at time t, then it is increasing at time t. However, increasing inflation does not necessarily imply acceleration.

Proof Outline

(a) Accelerating Inflation ⇒ Increasing Inflation
If:

$$\frac{d^2\pi}{dt^2} > 0$$

Then $\frac{d\pi}{dt}$ is increasing near t. If $\frac{d\pi}{dt}(t) \geq 0$, inflation is increasing. If it's zero or negative, it is turning.

(b) Increasing Inflation ⇒ Accelerating Inflation
Inflation can increase steadily or even slow its rise. That is:

$$\frac{d\pi}{dt} > 0, \quad \frac{d^2\pi}{dt^2} \leq 0$$

Conclusion

$$\text{Accelerating} \Rightarrow \text{Increasing}, \quad \text{but not vice versa.}$$

8 Final Thoughts

- Inflation is the *percentage* change in the price level — not just its derivative.
- It can fall even if prices rise.
- Differentiating between increasing and accelerating inflation is vital for policy.
- Derivatives and ratios provide rigorous tools for interpretation.

If you would like an interactive tool or code version of this article, feel free to ask.